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Purdue University

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John Warren Reese

A STUDY OF THE FLOW OF BULK MATERIAL IN BINS.

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A STUDY OF THE FLOW OF BULK MATERIAL IN BINS

A Thesis

Submitted to the Faculty

of

Purdue University

by

John Warren Reese, Jr.

In Partial Fulfillment of the

Requirements for the Degree

of

Master of Science

in

Industrial Engineering

May, 1954

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ABSTRACT

The major problem in handling bulk material in bins is flow stoppage, which is the hanging-up of material within the bin. Although there are generalizations as to why materials cease to flow, arch, or funnel, clear expression of the problem and the factors involved is lacking. One purpose of this thesis, therefore, is to study the background material and previous publications in an attempt to define the factors contributing to flow stoppage in a bin. In addition, an attempt will be made to isolate a particular factor to determine experimentally its contribution to flow stoppage.

Flow stoppage occurs in two common configurations, arching, which is the formation of a dome or span across the bin, and ratholing, which is a void passing up through the mass much like a rathole. The formation of a void in the area directly above the discharge gate and breaking through to the surface is a special case of ratholing known as "funneling".

Review of the literature resulted in a classification of the factors pertinent to flow stoppage as physical factors and design factors. The physical factors are properties of the material handled and of the confining materials. For the material handled the factors are inter-particle friction, moisture content, and compaction. The friction between the wall and the material handled, for any one material, will vary with the wall material. The plane of rupture is a hybrid between the inter-particle friction and the particle-to-wall friction. It defines that plane along which material flow will commence when under confinement. The design factors are structural, consisting of the size

of the discharge area, the cross-section of the bin, the wall inclination, and feeder lines.

The laws of semi-fluids as formulated by Janssen and by Airy express the influence of the physical factors upon the distribution of load forces within a bin. The vertical pressure in the bottom of the bin does not increase linearly as with fluids. The pressure increase is curvilinear up to a height equivalent to two and one-half diameters beyond which the increase is negligible. The lateral pressure follows the same performance, only to a lesser magnitude. These laws do not justify the occurrence of an arch or a funnel.

The theory of cohesive flow best explains the occurrence of an arch, while a modification of this concept explains funeling. In cohesive flow, the material transmits its weight to the side walls through a force network, comparable to a dome. In addition, the material strength is sufficient to support the center section of the dome. Under conditions of sufficient material strength and optimum transmission of load, the dome is capable of supporting the entire mass above it, thus resulting in an arch. If, however, the material strength is insufficient, the central section of the dome will collapse. When this occurs for each succeeding network throughout the bin, plug flow is obtained. The strength of a particular material is a function of compaction pressure, which varies with loading rate, the drop of the material, and bin configuration. For a particular system, the material strength may be determined, and knowing this, it is possible to design a discharge area of sufficient span to prevent bridging. The dimensions of the discharge may be determined by using Jenike's formula, $B > b_v b_v \frac{fc}{w} \sin 2\theta$.

The coefficients b_h and b_v are functions of the bin configuration, and the ratio f_c/w expresses the strength of the material.

There is some disagreement within the field as to the value of straight-side hoppers in alleviating flow stoppage. The majority opinion favors the use of straight sides whenever possible.

Two independent sections exist in a bin: the upper section and the hopper section. The loading in the upper section has very little effect upon the hopper loading. However, a maximum pressure area exists at the transition of these two areas and arching may occur at this location as well as in the hopper. The upper section customarily has vertical sides. The author reasoned that the force distribution in a potential arch of a section with inwardly sloping sides would have less stability. An experiment was designed to determine the effect of inwardly sloping sides upon the flow.

A wooden bin was constructed in such a manner that the angle of inclination of the two opposing sides could be varied. The front wall of the bin was plate glass which permitted an opportunity to observe the flow pattern. A cohesive and a free-flow material were individually cycled through the bin at various angles of inclination. The percent volume of discharge and the rate of discharge for each angle of inclination was determined. The original hypothesis that percent of discharge would increase as the inward slope increased was not sustained. It wasconcluded, based upon the bin and materials used in this experiment, that the slope angle of the bin side has no apparent effect of material flow either for cohesive or free-flow materials. For free-flow materials the discharge area controls the flow rate. For cohesive materials the discharge area determines the ability to flow.

A STUDY OF THE FLOW OF BULK MATERIAL IN BINS

INTRODUCTION

The time honored method of storing bulk materials has been in overhead containers known as bins, silos, elevators, or hoppers. Construction engineers are capable of designing these containers to withstand the loads involved, however, the formulas for designing for material flow are not available. It is common practice to load a bin with material only to find out later that the material will not flow free on demand. The material jams and must be dislodged with such instruments as sledge hammers and lances. This was bad enough when the bins were isolated units serving as raw material storage, but, to-day, they have become an integral part of the materials handling system in power plants, continuous chemical processing plants, food processing plants, steel processing plants, and on ad infinitum.

As a part of such a system the bin is usually required to provide material flow on demand or at some predetermined rate. When under these conditions the material jams, or hangs up or flows with restricted capacity, the entire system must be shut down while attempts are made to restore flow. In some continuous processing industries this means stopping all productive activity. It is difficult to conceive that modern engineering has been living with such an "ulcer", but such is the case. That a multi-million dollar plant can be designed to minute detail and still be plagued with such an occurrence is evidenced by Leggett[1] in the following report:

---

1. Leggett, R. F., "Clogging of Bituminous Coal in Bunkers," A.S.M.E. Trans., July 1947, pp. 525-533.

Canada's largest steam power station was constructed in 1942-1943 as a part of the extensive plant of Polymer Corporation Limited, government-owned synthetic rubber producer, adjacent to the St. Clair River about two miles south of Sarnia, Ontario. The Polymer installation is the only completely integrated synthetic-rubber plant in North America having a designed annual productive capacity of 37,000 tons of Buna-S and 6,000 tons of Butyl rubber.

The powerhouse is a building 286 ft. long, 173 ft. wide, and 120 feet high. The primary purpose of the plant is the production of process steam although power is also generated. Therefore, it is equipped with five Babcock and Wilcox boilers, with space for a future sixth unit, each capable of generating 308,000 lb. of steam per hour at 415 psi, at 650°F; power is generated at 6,600 volts, 60 cycles, by means of two-steam turbine units, rated as 11,000 kva at 0.90 pf, and one 5,000-kva unit at 0.80 pf.

Bituminous coal is at present used as fuel being delivered by water, stored adjacent to the station, and transported into the building by means of a 30-in. belt conveyor, after passage through a primary crusher installation. The coal is fed from bunkers, through automatic batch weighing devices, to pulverizing units and thence is blown into the combustion chambers. .... Capacity of one bunker is about 200 tons. Daily consumption of the station is about 800 tons, so that outside storage has to be provided for about 180,000 tons, to allow for seasonal delivery.

Shortly after the start of operations it was found that the coal did not always flow freely from the bunkers when the discharge gates were opened, even though the openings were 30 in. square in cross-section. Holes had been left in the bunker sides, just above the gates, for drainage purposes but they could not be used in attempts to loosen the coal. Eventually holes had to be cut in the discharge chutes to enable the operators on the operating floor to push long air lines into the coal and in this way disturb it sufficiently to cause it to resume its flow.

The measure was partially effective, but it was sometimes found that the coal would (literally) arch between the sides of the bunker some distance above the gates. At other times, a vertical hole about 2 to 3 feet diameter would form immediately above the gate opening, increasing in height until it revealed itself on the top surface of the stored coal. Accordingly it was necessary to station two men in the operating gallery above the bunkers, equipped with long poles, and eventually with long air jets, (which were found to be unusually effective), in order to keep the coal moving from the top. The men had to break down the

clogging, when it became evident, with their poles and jets. Frequently the coal could be seen to overhang, so firmly did it pack, even to the extent of 2 or 3 feet in a depth of not more than 10 feet.

PURPOSE

The purposes of this thesis are twofold:

1. To study the background material and previous publications in an attempt to define the factors contributing to flow stoppage in a bin.
2. To attempt to isolate a particular factor for the purpose of experimentally determining its contribution to flow-stoppage.

REVIEW OF LITERATURE

It has been shown by Leggett[2] how bulk materials will bridge or tunnel and one may estimate for a particular system the results of such an occurrence. To understand why this may occur one must become familiar with the factors involved when handling bulk materials. First, let us consider those items which are inherent in the material and its environment. These are defined as the physical factors. Later the design factors will be considered. These factors are those which may be controlled by design of the bin.

1. Physical Factors

Angle of Repose: When a bulk material such as sand is allowed to flow from overhead onto the ground, the sand will build up into a conical mass. The angle the conical surface makes with the horizontal is defined as the angle of repose. Assuming dry sand, the build-up occurs as the result of internal friction. This internal friction is inter-particle friction which is expressed as a decimal coefficient. However, it is more commonly referred to in terms of the slope of the mass, the angle of repose. The angle of repose is that angle whose tangent is the coefficient of inter-particle friction. It is the slope angle of the mass (ϕ). In the definition dry material is assumed, but this is seldom the case. Moisture content and degree of compaction greatly influence the angle of repose. There are several methods of determining the angle of repose, two of which are illustrated in Figure 1.

In Figure 1 (a) the test specimen is placed in the pan on the counter-weighted frame. Care must be taken to completely fill the pan

---

2. Ibid.

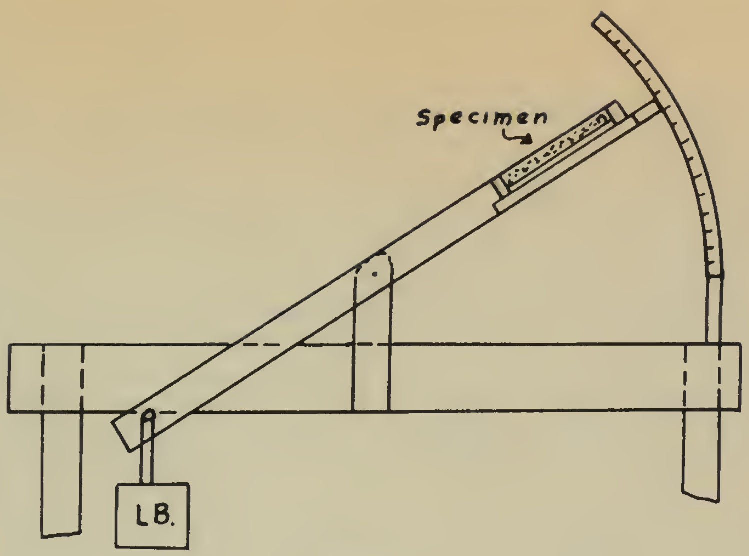
with the material to be tested. The frame is then inclined until the first particle motion observed, at this position record the angle of repose. For the system depicted in Figure 1(b) the test specimen is placed in the split box. A known vertical force is applied to the container. At the same time increasing equal and opposite lateral forces are applied to the upper and lower sections of the box. The lateral forces are increased until slippage occurs, and the lateral force is recorded. The coefficient of friction is:

$$\mu = \frac{\text{Vertical Force}}{\text{Lateral Force}} .$$

<u>Moisture Content</u>: The moisture content can greatly affect the performance of the bulk material. This was best demonstrated in a series of experiments handling pulverized coal conducted by Leggett[3] and in his summary of a paper by Guy.

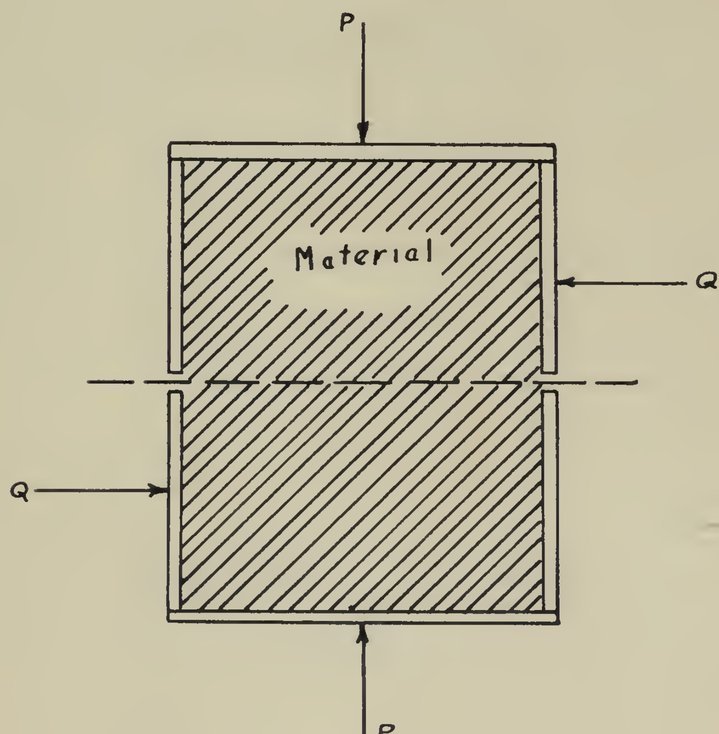
> Throughout the testing program it gradually became clear that the moisture content of the coal was a critical factor. As other possibilities were gradually eliminated, more attention was devoted to the quantity of water present in the coal. It was finally found, as the result of a large number of tests, that if the moisture content of the coal was less than 5 per cent, the coal would flow through the bunker exactly as had been intended in design. If, on the other hand, the moisture content was more than 6 per cent, clogging would take place.
>
> In seeking for an explanation of this rigid limitation, it was thought possible that the moisture content indicated might be related to that at which all the voids between the coal particles were just full of water, any further addition resulting in some excess of water. In soil-mechanics work, this value is known as the "optimum moisture content," it is determined by a technique known as the Proctor test after the engineer who first suggested it. In this test, samples of soil are mixed with increasing quantities of water, compacted in a small cylinder under identical conditions, and weighed. The density is found to increase until the optimum moisture content is passed, after which the

3. Ibid.



(a)

Fig. 1



(b)

Fig. 1

Apparatus for Testing the Angle of Repose

(From Ketchum's, Walls, Bins and Grain Elevators)

density decreases due to the displacement of solid material by water.

Therefore a sample of coal was subjected to the Proctor test ....

In view of the wide difference between the critical moisture range of from 5 to 6 percent and the optimum moisture content of about 16 percent, it became clear that free moisture was not a determining factor in the problem. That the water making up the 5 to 6 percent at the critical range was not free water was easily demonstrated by leaving a tall glass tube filled with coal at this moisture range standing vertically for several weeks. Moisture contents at the top, middle, and bottom of the tube remained unchanged.

The solution thus indicated, by analogy with soil properties, was a very simple one. The small quantity of water represented by the critical quantity must be just sufficient to form absorbed moisture films around the coal particles making up the mass. The angular shape and the small quantity of very small coal particles are together sufficient to create internal adhesion or "apparent cohesion" which is the cause of the sticking together of the coal. The action is of course assisted by the almost ideal grading of the coal, and by the slight amount of "packing" which results from the deposition of the coal in the bunkers by dropping it from the overhead belt conveyor.

Compaction: Compaction may occur in confinement or in open volumes of bulk material. It is a function of the loading rate, drop of the material, and jumbling as in railroad cars. It is also greatly affected by the particle gradation of the bulk material.

Thus far the preceding factors, internal friction, moisture content and compaction have been defined in terms of their individual characteristics. Under actual handling conditions it is difficult to isolate these factors; in reality they are interdependent. Moisture content to a point will tend to increase the angle of repose by introducing a cohesion effect. Beyond a certain critical moisture percentage the angle of repose will then decrease until, under extreme conditions,

the reaction would be that of a fluid. Compaction will also increase the angle of repose. In moisture-free material the increase is noticeable, however, with a slight moisture content the angle of repose will rapidly reach 90° or vertical suspension. The interrelation of these factors was amply expressed by Wolf and von Hohenleiten[4] while investigating the flow of coal in chutes at the Riverside Plant of the Consolidated Gas, Electric Light and Power Company of Baltimore. In this work Wolf and von Hohenleiten present quantitative measurements of these physical factors in coal. These measurements and pertinent extracts are presented here to illustrate the degree of interaction of these properties. The measurements are not adaptable to other materials or necessarily other gradations of coal.

> One of the properties which governs the gravity flow of coal is the static angle of repose which is influenced by both moisture content and degree of compacting. for the determination of this static angle of repose, a 12-in. by 12-in. x 12-in. metal box with one side hinged at the bottom was used. The box was first filled with loose coal, then the hinged side was dropped, and, after all loose coal had fallen away, the angle of the cleavage plane was observed. The box was then filled with coal and vibrated until full compactness was reached. Again the side was dropped and the cleavage angle noted. This test was repeated for moisture contents ranging from one per cent to 18 per cent and the results are shown in Figure (2). With fully compacted coal, the angle of reposes increases rapidly and reaches 90° at 3.6 percent moisture. Consequently, at this and higher moisture contents, ratholing can occur in hoppers and bunkers. With an increase in moisture content above 3.6 percent, it was possible to undercut the coal. At higher moisture ranges (above 13 percent), the wet mass of fully compacted coal had a tendency to bulge out. It may be stated, therefore,

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4. Wolf, E. F. and H. L. von Hohenleiten, "Experimental Study of the Flow of Coal in Chutes at Riverside Generating Station," Transactions of the A.S.M.E., October, 1945.

that the angle of repose for these conditions exceeds 90°, or, wording it differently, reaches negative values. From the investigations mentioned, it is evident that it is important to use the steepest possible angles, preferably 75° or more, in the design of coal chutes if moisture contents higher than 3 percent are to be encountered. All of the tests have indicated that the degree of compactness or density of the coal greatly affects the flow characteristics.

The 1-cu-ft box used in previous tests was filled repeatedly with coal of various moisture content, and the weight per cubic foot checked. These weights have been plotted for loose and fully compacted coal in Figure (3). Starting from one percent moisture content, the density of loose coal decreases until it reaches its lowest value at approximately 7 percent. Above 7 percent, it rises until at 15.75 percent it again reaches the same density as at 1 percent moisture and continues to rise. Some other granular materials, for instance, sand, show this same phenomenon of a point of minimum density at an intermediate moisture content.

Friction of Particle on the Bin Wall. In the study of fluid flow one finds that friction does not exist between the fluid and the pipe. There is a surface coating of the fluid on the pipe which is stagnant. The shear occurs between this layer and the succeeding layer of fluid. This is not the case in handling bulk materials. There is friction between the wall and the confined material. The magnitude of this friction varies with the wall material and with the confined material. Jamieson[5] investigated the variance in the coefficient of friction by measuring the angle of slide of various wall materials on wheat. Ketchum[6] reports the results of this investigation, and as may be seen in Table 1, for the common wall materials there is a considerable deviation.

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5. Ketchum, Milo S., "Walls, Bins, and Grain Elevators," 3rd edition, McGraw-Hill Book Company, Inc., New York, N. Y., 1929, page 339.

6. Ibid.

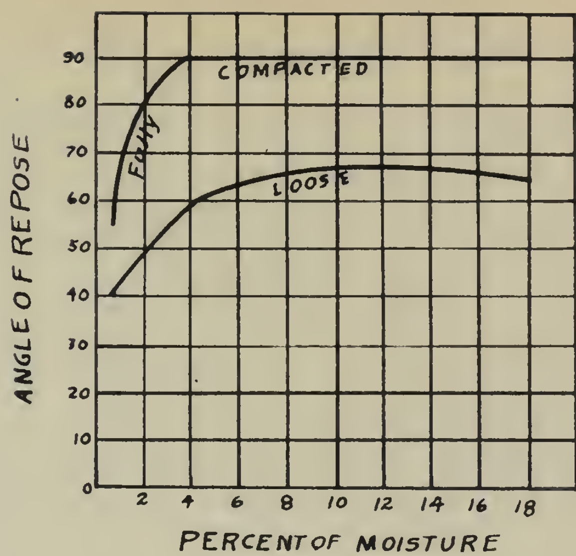

Fig. 2

Effect of Moisture Content of Coal

on Static Angle of Repose

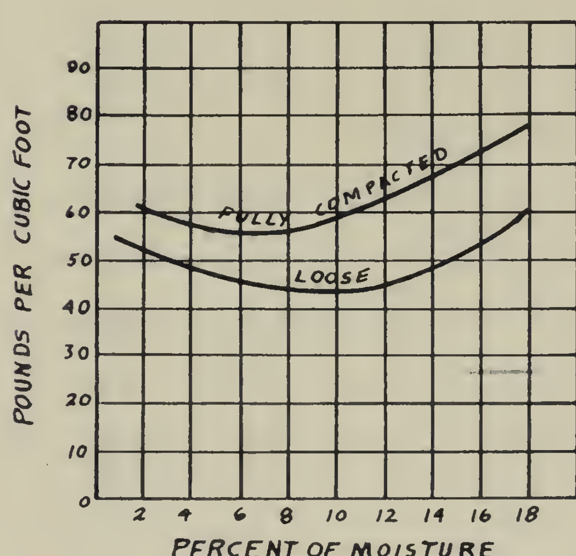

Fig. 3

Effect of Moisture Content on Density of Coal

(From Transactions of A.S.M.E., October, 1945)

TABLE 1

COEFFICIENTS OF FRICTION AND ANGLE OF REPOSE OF WHEAT
WHEAT WEIGHING 50 LBS. PER CUBIC FOOT
ANGLE OF REPOSE Ø - 28 Degrees

Materials	Coefficient of Friction
Wheat on wheat	0.532
Wheat on steel through plate bin.................	0.468
Wheat on steel flat plate, riveted and tie bars ...	0.375 to 0.400
Wheat on steel cylinders, riveted	0.365 to 0.375
Wheat on cement concrete, smooth to rough	0.400 to 0.425
Wheat on tile or brick, smooth to rough............	0.400 to 0.425
Wheat on cribbed wooden bin	0.420 to 0.450

The complexity of this factor was demonstrated by Wolf and von Hohenleiten[7] when they studied the effect of moisture content upon the slide angle of coal. It was found that within the range of moisture content observed, one per cent to sixteen per cent, the following occurred: Up to eight per cent moisture the slide angle increased, beyond this percentage the tendency for the slide angle was to decrease. Perhaps qualification of the last statement is required. These tests were run on a series of different materials. In the range 0-8 per cent moisture all materials increased in slide angle. From 8-16 per cent some materials, glass, aluminum, and stainless steel took a sharp turn downward. The other materials, rubber, pyralin, galvanized steel and sheet steel, tested in the 8-16 range, maintained a relatively constant angle of slide. In the case of the glass, aluminum, and stainless steel it was proposed that the moisture content above 8 per cent was sufficient to permit the formation of a surface film of moisture

7. Wolf, E. F., and H. L. von Hohenleiten, "Experimental Study of the Flow of Coal in Chutes at Riverside Generating Station", Transactions of the A.S.M.E., October 1945.

which allowed the entire sample to slide as one mass.

The above tests were performed on free specimens with only surface contact. Further runs were made in metal chutes and in cylindrical tubes in an attempt to correlate the behavior of free coal and confined coal. Here again it was found that a maximum slide angle was reached followed by a decline with an increase in moisture content. Thus when one considers particle to wall friction he must consider the wall material, the type of filler, the moisture content and degree of confinement.

Plane of Rupture: To introduce this factor, it is desirable to consider Coulomb's Theory of Pressure on Retaining Walls. For a retaining wall backed with material free of surcharge (Figure 4) Coulomb[8] defines a wedge of material which exerts a maximum thrust upon the wall. This wedge is bounded by the back surface of the wall, the free surface of the material and a plane through the mass. This plane is called the Plane of Rupture. If we consider for a moment the wedge as a body the plane of rupture is that surface upon which the body would slide. The weight of this body is sufficient to overcome the shear strength existing between the particles along that plane. If this were a free mass of material upon the ground, the angle of the plane would correspond to the angle of repose.

When material is confined by a retaining wall the angle of repose is not a limiting factor. Further confinement as in a bin also illustrates the complexity of the plane of rupture. Leygue[9] very aptly

---

8, Cain, William, "Earth Pressure, Walls, and Bins", John Wiley & Sons, Inc., New York, N.Y.

9. Ibid.

demonstrated the plane of rupture in a series of experiments using stratified material between glass plates. The results of these experiments were reported by Cain[10] and are presented as an excellent demonstration of this factor. (Figure 5). As may be observed in the case of the movable wall, when the wall is withdrawn the material fills the void, sliding along the plane of rupture. The same occurs with the tilted wall.

In Airy's[11] solution of the stresses in bins (to be presented later) it is developed mathematically that the plane of rupture is a function of the angle of repose and the coefficient of friction of material to wall. This relation is expressed as:

$$\tan x = \mu + \sqrt{\mu + \frac{1 + \mu^2}{\mu + \mu'}}$$

where x = angle of inclination of plane of rupture

μ = coefficient of particle friction

μ' = coefficient of particles on wall

2. Law of Semi-Fluids

Bulk materials fall in a category between solid and liquids. They are known as semi-fluids, because, as has already been illustrated, they do not act like solids nor do they follow the behavior of solids. Perhaps the most distinguishing feature of bulk materials is the pressure and load distribution in confinement. At one time grains and coal were stored in bins designed upon hydrostatic prin-

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10. Ibid 2.

11. Ibid 1, page 316.

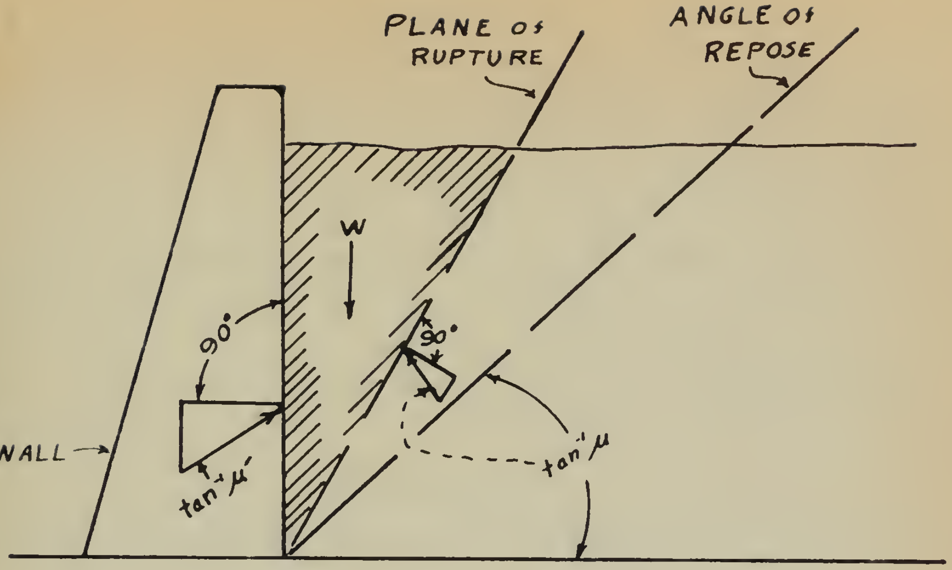


Fig. 4

Wedge of Maximum Pressure on Retaining Wall

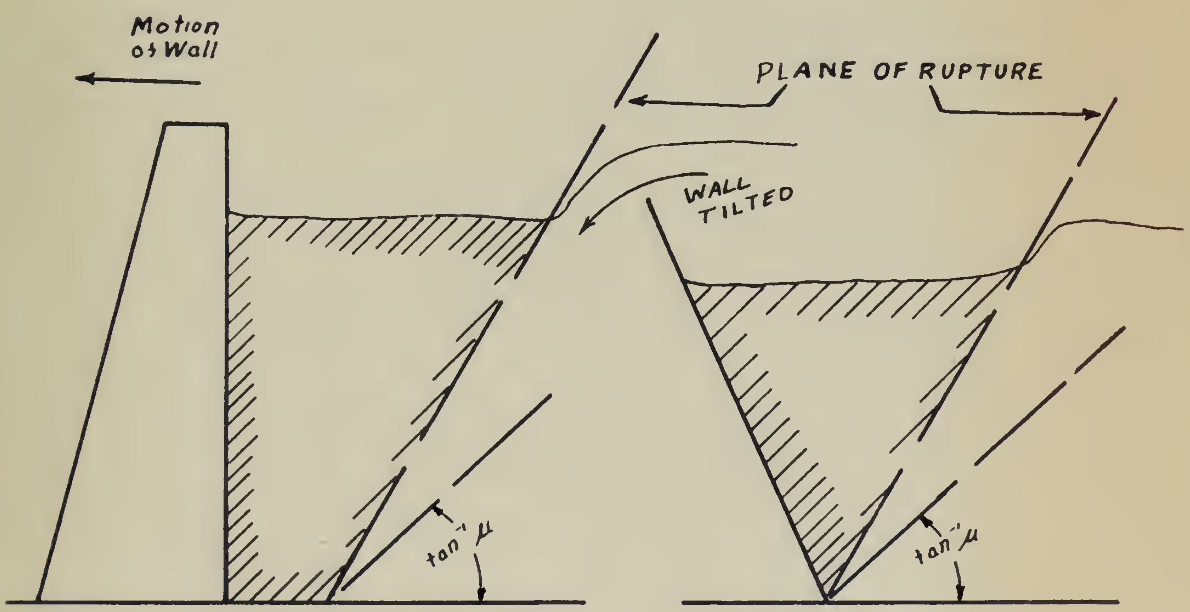


Fig. 5

Movement of Material Along Plane of Rupture

(Courtesy of Chemical Engineering)

ciples but after several dramatic failures other explanations were sought. There are two classical solutions to this problem, which brought to light the fallacy in the hydrostatic approach, one by Janssen and the other by Airy. Both of these are contained in Ketchum's Walls, Bins, and Grain Elevators[1]. These solutions, particularly Janssen's, have been the basis for bin design since the turn of the century. They are presented here as developed in Ketchum's book:

Nomenclature:

ϕ = the angle of repose of the grain;

ϕ' = the angle of friction of the grain on the bin walls;

μ = $\tan \phi$ = coefficient of friction of grain on grain;

μ' = $\tan \phi'$ = coefficient of friction of grain on the bin walls;

x = angle of rupture;

w = weight of grain in lbs. per cu. ft.;

V = vertical pressure of the grain in lbs. per sq. ft.;

L = lateral pressure of the grain in lbs. per sq. ft.;

A = area of bin in sq. ft.;

U = circumference of bin in feet;

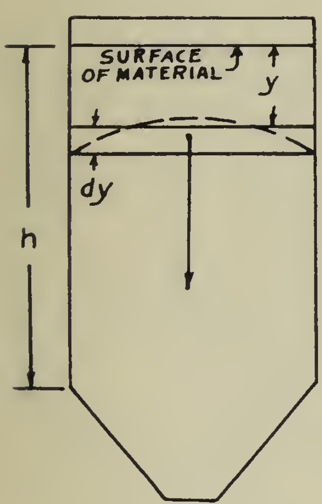
R = A/U hydraulic radius of bin.

---

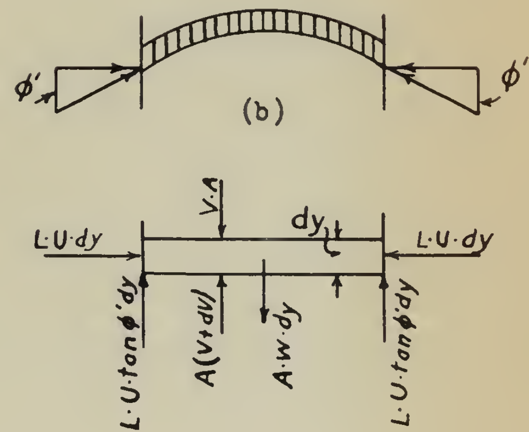
1. Ibid.

Janssen's Solution:[12] The bin in (a) Figure 6 has a uniform area A, a constant circumference U, and is filled with grain weighing w per unit of volume, and having an angle of repose ϕ. Let V be the vertical pressure, and L be the lateral pressure at any point, both V and L being assumed as constant for all points on the horizontal plane. (More correctly V and L will be constant on the surface of a dome as in (b).)

The weight of the grain between the sections of y and y + dy = $A \cdot w \cdot dy$; the total frictional force acting upwards at the circumference will be $L \cdot U \cdot \tan \phi' \cdot dy$; the total perpendicular pressure on the upper



(a)



(c)

Fig. 6

Distribution of Forces in a Bin

surface will be = $V \cdot A$; and the total pressure on the lower surface will be = $(V + dV)A$.

Now these vertical pressures are in equilibrium, and

$$V \cdot A - (V + dV)A + A \cdot w \cdot dy - L \cdot U \cdot \tan \phi' \cdot dy = 0,$$

and

$$dV = \left(w - L \cdot \tan \phi' \frac{U}{A}\right) dy \qquad (1)$$

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12. Ibid 5, page 307.

Now in a granular mass, the lateral pressure at any point is equal to the vertical pressure times k, a constant for the particular grain, and

$$L = k.V$$

Also let $\frac{A}{U}$ = R (the hydraulic radius), and $\tan \phi' = \mu'$.

Substituting the above in (1) we have

$$dV = (w - \frac{k.V}{R}\mu')dy$$

Now let

$$\frac{k.\ \mu'}{R} = n \qquad (2)$$

and

$$\frac{dV}{w - n.V} = dy \qquad (3)$$

Integrating (3) we have

$$\log (w - n.V) = - n.y + C \qquad (4)$$

Now if y = 0, then V = 0, and C = log w, and (4) reduces to

$$\log \left(\frac{w - n.V}{W}\right) = - n.y,$$

and

$$\frac{w - n.V}{w} = \frac{1}{e^{n.y}} = e^{-n.y}$$

where e is the base of the Naperian system of logarithms. Solving for V, we have

$$V = \frac{w}{n}\ (1 - e^{-n.y})$$

Substituting the value of n from (2) we have

$$V = \frac{w.R}{k \cdot \mu'}\ (1 - e^{\frac{-k \cdot \mu' y}{R}})$$

Now if h be taken as the depth of the grain at any point, we will have

$$V = \frac{w \cdot R}{k \cdot \mu'} \left(1 - e^{\frac{-k \cdot \mu' \cdot h}{R}}\right)$$

Also since $L = k \cdot V$,

$$L = \frac{w \cdot R}{\mu'} \left(1 - e^{\frac{-k \cdot \mu' \cdot h}{R}}\right) \tag{5}$$

Now if w is taken in lbs. per cu. ft., and R in feet, the pressure will be given in lbs. per square foot.

The walls of a deep bin carry the greater part of the weight of the contents of the bin. The total weight carried by the bin walls is equal to the total pressure P, of the grain on the bin walls, multiplied by the coefficient of friction μ' of the grain on the bin walls.

From formula (5) the unit pressure on a unit at a depth y will be

$$L = \frac{w \cdot R}{\mu'} \left(1 - e^{-k \cdot \mu' \cdot y/R}\right) \tag{6}$$

and the total lateral pressure fór a depth y, per unit of length of the perimeter of the bin, will be

$$P = \int_0^y L \cdot dy = \int_0^y \frac{w \cdot R}{\mu'} \left(1 - e^{-k \cdot \mu' \cdot y/R}\right) dy$$

$$= \frac{w \cdot R}{\mu'} \left[y - \frac{R}{k \cdot \mu'} + \frac{R}{k \cdot \mu'} \cdot e^{-k \cdot \mu' \cdot y/R} \right] \tag{7}$$

Now the last term in (7) is very small and may be neglected for depths of more than two diameters, and

$$P = \frac{w \cdot R}{\mu'} \left[y - \frac{R}{k \cdot \mu'} \right] \text{(approx.)} \tag{8}$$

The total load per lineal foot carried by the side walls of the bin will be

$$P \cdot \mu' = w \cdot R \left[y - \frac{R}{k \cdot \mu'} \right] \text{(approx.)} \tag{9}$$

For the total weight of grain carried by the side walls multiply (9) by the length of the circumference of the bin.

Airy's[13] Solution: There are two cases depending upon the ratio of width or diameter of the bin to depth of grain: Case I, Shallow Bins, where the plane of rupture cuts the surface of the grain within the bin; and Case II, Deep Bins, where the plane of rupture intersects the side of the bin wall.

Case I. Shallow Bins

To find the pressure on the sides and bottom of a bin, when the depth of grain in the bin is such that the plane of rupture passes out of the grain before it meets the opposite side of the bin. Let CABD be a vertical section of a bin, Fig. 7, and CD the surface of the grain.

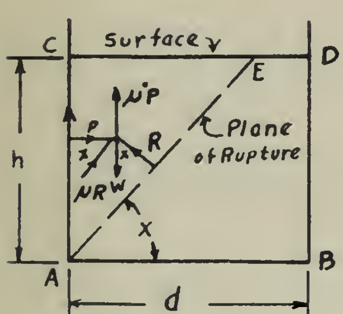

Fig. 7

Shallow Bin

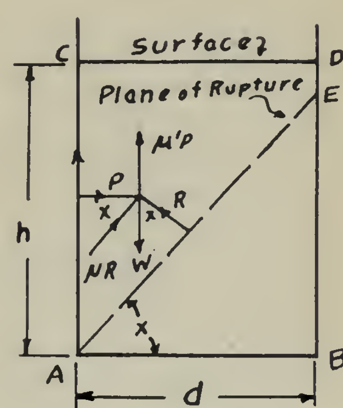

Fig. 8

Deep Bin

Let AE be the plane of rupture. Then ACE is the wedge-shaped mass of grain which causes maximum pressure against the side AC, and O its center of gravity. Let W be the weight of ACE, 1 foot thick, P the pressure against the side AC, $\mu' \cdot P$ the friction between the grain and the side AC, acting in the direction AC, μ' being the coefficient of friction, R the pressure of the mass ACE on the plane of separation AE. $\cdot\ \mu \cdot R$ the friction between the grain along the plane of separation AE, μ being the coefficient of friction, h the depth of grain AC in feet, x the angle EAB which the plane of rupture makes with the horizontal, and w the weight of one cubic foot of the grain in pounds.

For convenience, the forces are all drawn as if acting at O in their proper directions; and resolving the forces that support the

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13. Ibid 1.

mass ACE parallel and perpendicular to AE,

$$\mu \cdot R + P \cdot \cos x = (W - \mu' \cdot P) \sin x$$

$$R - P \cdot \sin x = (W - \mu' \cdot P) \cos x$$

whence

$$\mu = \frac{(W - \mu' \cdot P) \sin x - P \cdot \cos x}{(W - \mu' \cdot P) \cos x + P \cdot \sin x}$$

and from this by reduction,

$$P = W \frac{\tan x - \mu}{1 - \mu \cdot \mu' + (\mu + \mu') \tan x}$$

But

$$W = w \times \frac{1}{2} AC \times CE = w \times \frac{1}{2} h \times \frac{h}{\tan x} = w \times \frac{h^2}{2 \tan x}$$

$$P = w \times \frac{h^2}{2 \tan x} \times \frac{\tan x - \mu}{1 - \mu' \cdot \mu + (\mu + \mu') \tan x}$$

$$= w \times \frac{h^2}{2} \frac{\tan x - \mu}{(1 - \mu \cdot \mu') \tan x + (\mu + \mu') \tan^2 x}$$

To find the value of x that causes P to be a maximum, place

$$\frac{dP}{dx} = 0$$

Now

$$\frac{dP}{dx} = w \times \frac{h^2}{2} \left\{ \frac{1}{\cos^2 x} \left[(1 - \mu \cdot \mu') \tan x + (\mu + \mu') \tan^2 x \right] - (\tan x - \mu) \right.$$

$$\left. \times \left[\frac{1}{\cos^2 x} (1 - \mu \cdot \mu') + 2 \tan x \times \frac{1}{\cos^2 x} \times (\mu + \mu') \right] \right\}$$

all divided by

$$\left\{ (1 - \mu \cdot \mu') \tan x + (\mu + \mu') \tan^2 x \right\}^2,$$

therefore

$$(1 - \mu \cdot \mu') \tan x + (\mu + \mu')\tan^2 x - (1 - \mu \cdot \mu')(\tan x - \mu)$$

whence by reduction, $- 2 \tan x (\tan x - \mu)(\mu + \mu') = 0$

$$\tan x = \mu + \sqrt{\mu \times \frac{1 + \mu^2}{\mu + \mu'}}$$

since the part under the radical is greater than μ. If this value of tan x is substituted in the expression for P, the lateral pressure of the wedge ACE, 1 foot thick, will be obtained; and this result multiplied by the horizontal circumference of the bin in feet, gives the total pressure on the sides of the bin. This, multiplied by μ', gives the vertical sustaining force of the side friction; and the pressure on the bottom is the total weight of the grain less the vertical sustaining force of the friction. Thus the fundamental equations for Case I are:

$$\tan x = \mu + \sqrt{\mu \times \frac{1 + \mu^2}{\mu + \mu'}} \tag{10}$$

$$P = w \times \frac{h^2}{2 \tan x} \times \frac{\tan x - \mu}{1 - \mu \cdot \mu' + (\mu + \mu') \tan x} \tag{11}$$

Substituting tan x in (10) in (11)

$$P = \frac{1}{2} w \cdot h^2 \left[\frac{1}{\sqrt{\mu(\mu + \mu')} + \sqrt{1 + \mu^2}} \right]^2 \qquad (12$$

To calculate the unit pressure, L, at any depth, h = y, differentiate P in (12) with respect to the depth, h = y

$$L = w \cdot y \left[\frac{1}{\sqrt{\mu(\mu + \mu')} + \sqrt{1 + \mu^2}} \right]^2$$

The vertical pressure at any point in the bin will then be

$$V = L/k$$

Case II. Deep Bin

To find the pressure on the sides and bottom of a bin, when the depth of grain is such that the plane of rupture meets the opposite side of the bin within the mass of grain. Let d be the breadth of the bin, Figure 8. Then, as in Case I,

$$P = W \times \frac{\tan x - \mu}{1 - \mu \cdot \mu' + (\mu + \mu') \tan x}$$

But

$$W = w \times 1 \times \text{area AEDCA} = w \times CD \times \frac{AC + DE}{2}$$

$$= w \times \frac{d}{2} (2h - d.\tan x)$$

$$P = w \times \frac{d}{2} (2h - d.\tan x) \times \frac{\tan x - \mu}{1 - \mu \cdot \mu' + (\mu + \mu') \tan x}$$

or

$$P = w \times \frac{d}{2} \left\{ 2h \times \frac{\tan x - \mu}{1 - \mu \cdot \mu' + (\mu + \mu') \tan x} \right.$$

$$\left. - d \times \frac{\tan^2 x - \mu \cdot \tan x}{1 - \mu \cdot \mu' + (\mu + \mu') \tan x} \right\}$$

To find the value of x which makes P a maximum, place $\frac{dP}{dx} = 0$.

Now

$$\frac{dP}{dx} = w \times \frac{d}{2} \left\{ 2h \left[\frac{1}{\cos^2 x} (1 - \mu \cdot \mu') + \frac{1}{\cos^2 x} (\mu + \mu') \tan x \right. \right.$$

$$\left. - \frac{1}{\cos^2 x} (\mu + \mu') \tan x + \frac{1}{\cos^2 x} (\mu + \mu') \mu \right]$$

$$- d \left[\left(2 \tan x \times \frac{1}{\cos^2 x} - \mu \times \frac{1}{\cos^2 x} \right) \right.$$

$$\left\{ 1 - \mu \cdot \mu' + (\mu + \mu') \tan x \right\}$$

$$- \frac{1}{\cos^2 x} (\mu + \mu')(\tan^2 x - \mu' \tan x) \Big] \Big\}$$

all divided by

$$\Big\{ 1 - \mu\cdot\mu' + (\mu + \mu') \tan x \Big\}^2$$

so that

$$2h (1 + \mu^2) - d\Big\{ \tan^2 x (\mu + \mu') + 2 \tan x (1 - \mu\cdot\mu')$$

$$- \mu(1 - \mu\cdot\mu')\Big\} = 0,$$

which by reduction gives finally

$$\tan x = \sqrt{\frac{2h}{d} \times \frac{1 + \mu^2}{\mu + \mu'} + \frac{1 + \mu^2}{\mu + \mu'} \times \frac{1 - \mu\cdot\mu'}{\mu + \mu'}} - \frac{1 - \mu\cdot\mu'}{\mu + \mu'}$$

Substituting this value of tan x in the expression for P, the maximum pressure on the side of the bin of the wedge-shape mass AEDCA, 1 foot thick, is obtained; and the pressure on the sides and bottom of the bin can be deduced as before. Thus the fundamental equations for Case II are:

$$\tan x = \sqrt{\frac{2h}{h} \times \frac{1 + \mu^2}{\mu + \mu'} + \frac{1 + \mu^2}{\mu + \mu'} \times \frac{1 - \mu\cdot\mu'}{\mu + \mu'}} - \frac{1 - \mu\cdot\mu'}{\mu + \mu'} \qquad (13)$$

$$P = w \times \frac{d}{2} \times (2h - d\cdot \tan x) \times \frac{\tan x - \mu}{1 - \mu\cdot\mu' + (\mu + \mu')\tan x} \qquad (14)$$

Substituting tan x in (13) in (14)

$$P = \frac{1}{2} w\cdot d^2 \left[\frac{\sqrt{\frac{2h}{d} (\mu + \mu') + 1 - \mu\cdot\mu'} - \sqrt{1 + \mu^2}}{\mu + \mu'} \right]^2 \qquad (15)$$

To calculate the unit pressure, L, at any depth, h = y, differentiate P in (15) with respect to the depth, h = y

$$L = \frac{\delta . P}{\delta\ y} = \frac{w \cdot d}{u + u'} \quad 1 - \frac{\sqrt{1 + \mu^2}}{\sqrt{\frac{2h}{d}(\mu + \mu') + 1 - \mu \cdot \mu'}} \qquad (16)$$

The vertical pressure at any point in the bin will be

$$V = L/k$$

Of the two solutions presented, Janssen's is the most popular, mainly because it is the easiest to work with. It followed closely the results of experimental data obtained, and there were many opportunities for verification around the turn of the century. Ketchum[14] devotes a chapter to the discussion of experiments conducted on full size and scale bins. These experiments were all concerned with measuring the pressures existing in bins. In general, they upheld Janssen's Solution as shown in Figure 9. There is some disagreement on the effect upon pressure of a moving mass. One experimenter, Prante[15], obtained lateral pressures four times static pressure with moving wheat. The majority finding, however, was in the nature of a 10 to 15 per cent increase over static conditions. It was further found that there is a difference in pressure between that measured while loading and the value obtained while unloading.

The experimental technique was fairly standard throughout. Two hydraulic type pressure transmission gages were used, one on the

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14. Ibid. 1

15. Ibid.1

16. Ibid 1, page 336

bottom of the bin in a horizontal position to measure the bottom pressure and the other on the bottom side in a vertical position to measure the side pressure. The size of the pressure area was varied in several cases to see if the area of sensitivity was critical. It was found, however, that the unit pressure determined on the small areas closely followed those measured on the large area.

An interesting point was brought out by Jamieson[16] when he described the relationship of the percentage of wheat carried by the bottom and sides of a model bin. As may be seen in Figure 10 the side carries an increasing percentage as the height of material in the bin increases.

The location of the discharge area proved to be an effective factor in the distribution of pressures. When the area was off-set from center, in particular when located adjacent to one side the pressure on the opposite wall increased above that normally obtained.

By definition, K is the ratio of the lateral to vertical pressure. It was found that this factor was not a constant. It varied in a bin for different depths of the material, being greater for the small depth. Further variation occurs when using different materials and bin types.

The preceding statements are interpretations of the experiments reported by Ketchum[17]. Because of their importance in the understanding of material flow in bins they are repeated here.

It might be stated that Ketchum, in collaboration with Williams

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16. Ibid 1, page 336.
17. Ibid 1.

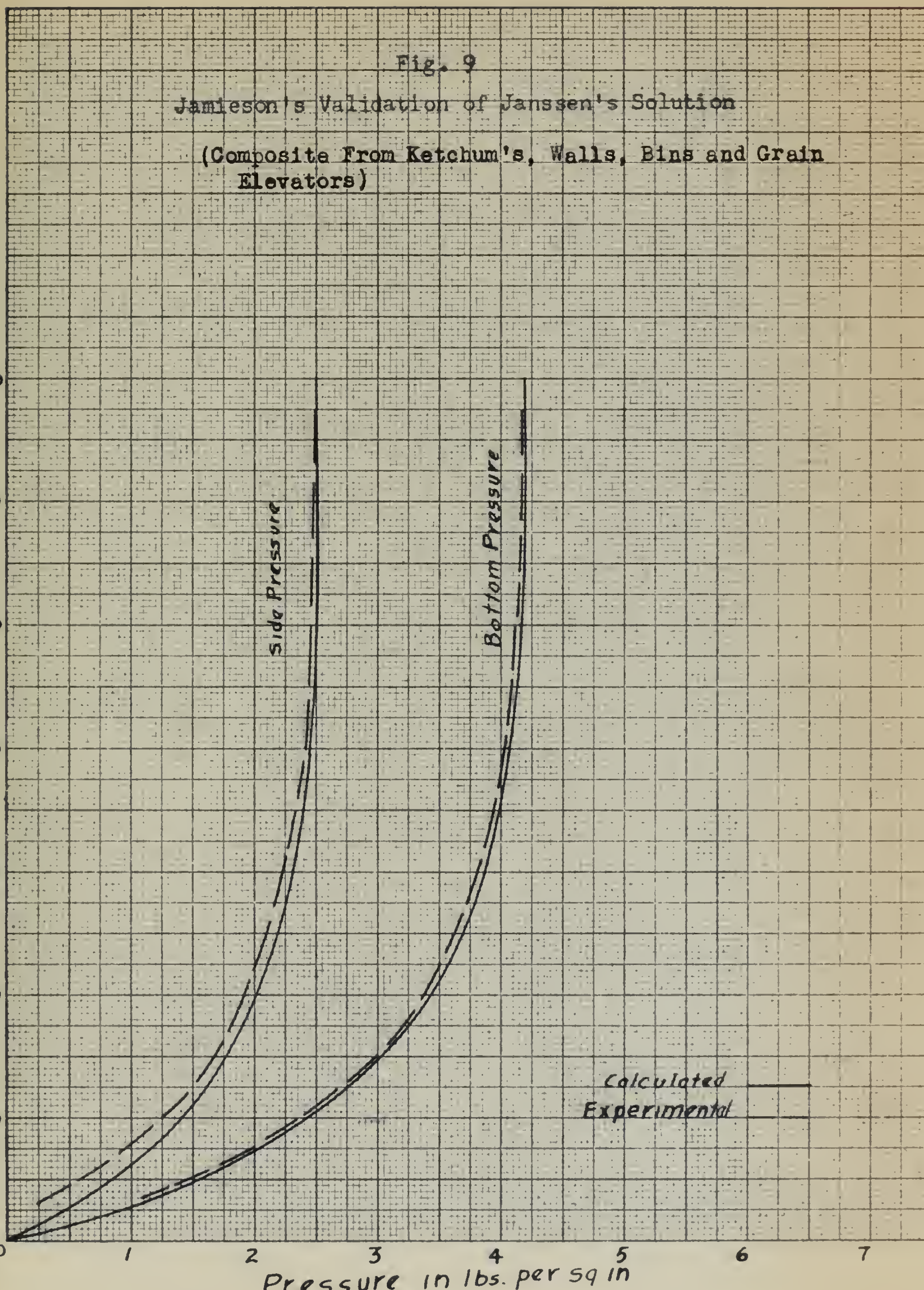
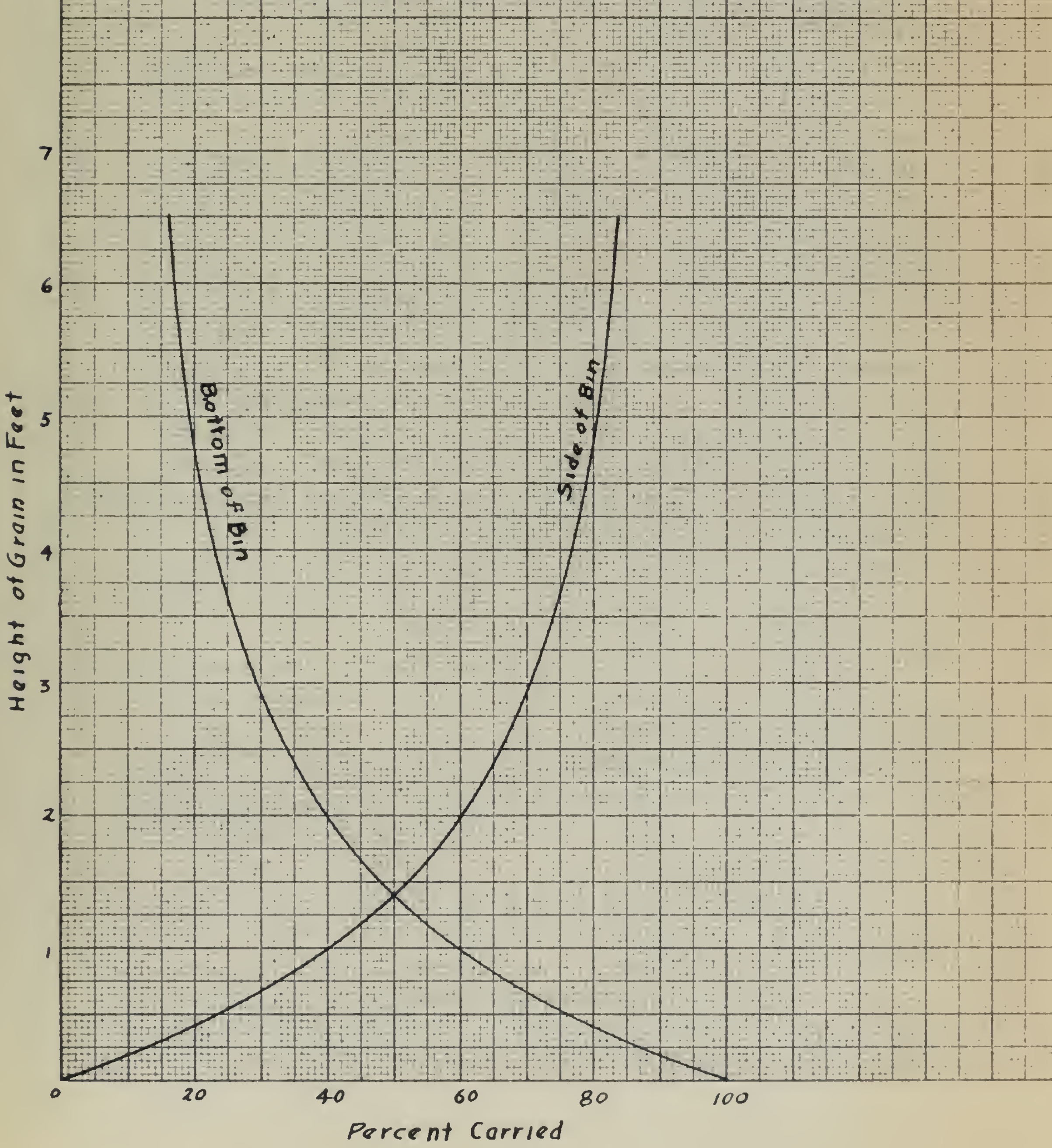


Fig. 9
Jamieson's Validation of Janssen's Solution
(Composite From Ketchum's, Walls, Bins and Grain Elevators)
Side Pressure
Bottom Pressure
Calculated
Experimental
Height of Grain in Feet
Pressure in lbs. per sq in
0
10
20
30
40
50
60
70
1
2
3
4
5
6
7

Fig. 10

Percentage of Material Carried by Bottom and Sides in a Model Bin

(From Ketchum's, Walls, Bins and Grain Elevators)



verified these results in experiments conducted at the University of Colorado.

Conclusions:

1. The pressure of grain on bin walls and bottoms follows a law (which for convenience will be called the law of "semi-fluids"), which is entirely different from the law of the pressure of fluids.

2. The lateral pressure of grain on bin walls is less than the vertical pressure (0.3 to 0.6 of the vertical pressure, depending on the grain, etc.) and increases very little after a depth of 2-1/2 to 3 times the width of diameter of the bin is reached.

3. The ratio of lateral to vertical pressures, k, is not a constant, but varies with different grains and bins. The value of k can only be determined by experiment.

4. The pressure of moving grain is very slightly greater than the pressure of grain at rest (maximum variation for ordinary conditions is probably 10 per cent).

5. Discharge gates in bins should be located at or near the center of the bin.

6. If the discharge gates are located in the sides of the bins, the lateral pressure due to moving grain is decreased near the discharge gate and is materially increased on the side opposite the gate (for common conditions this increased pressure may be two to four times the lateral pressure of grain at rest).

7. Tie rods decrease the flow but do not materially affect the pressure.

8. The maximum lateral pressures occur immediately after filling, and are slightly greater in a bin filled rapidly than in a bin filled slowly. Maximum lateral pressures occur in deep bins during filling.

9. The calculated pressures by either Janssen's or Airy's formulas agree very closely with actual pressures.

10. The unit pressures determined on small surfaces agree very closely with unit pressures on large surfaces.

11. Grain bins designed by the fluid theory are in many cases unsafe as no provision is made for the side walls to carry

the weight of the grain, and the walls are crippled.

12. Calculation of the strength of wooden bins that have been in successful operation shows that the fluid theory is untenable, while steel bins designed according to the fluid theory have failed by crippling the side plates.

Janssen's[18] solution was again validated in a series of experiments conducted by Rudd[19]. This time a limitation of the solution was brought to light. It became apparent that although the solution was valid for vertical sided bins it was not valid for slope sided bins. Rudd working under the auspices of the Richardson Scale Company has been investigating the flow of materials in bins. In his earlier experiments he attempted to determine the general applicability of Jannssen's solution. This was done by running tests in small sloped and straight-sided hoppers. The materials used were polystyrene pellets, Portland cement and dry mash feed. A pressure diaphragm was located under the discharge area to measure the bottom pressure. The empirical results for the straight sided circular hoppers coincided with the theoretical curve as may be seen in Figure (11). This figure also shows the deviation obtained in using a 5° and 20° slope-sided hopped. Rudd[20] interprets the data as follows:

"Results of the tests with the circular, straight-sided hopper checked with theoretical values obtained from Jannssen's equation. But they also indicated that Jannssen's equation did not apply for slope-sided hoppers."

It was felt by Rudd[21] that a more general equation could be formulated

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18. Ibid.
19. Rudd, J. K., "How Does Material Flow From a Bin?" <u>Milling Production</u>, January, 1954, Vol. 19, page 5.
20. Idem.
21. Idem.

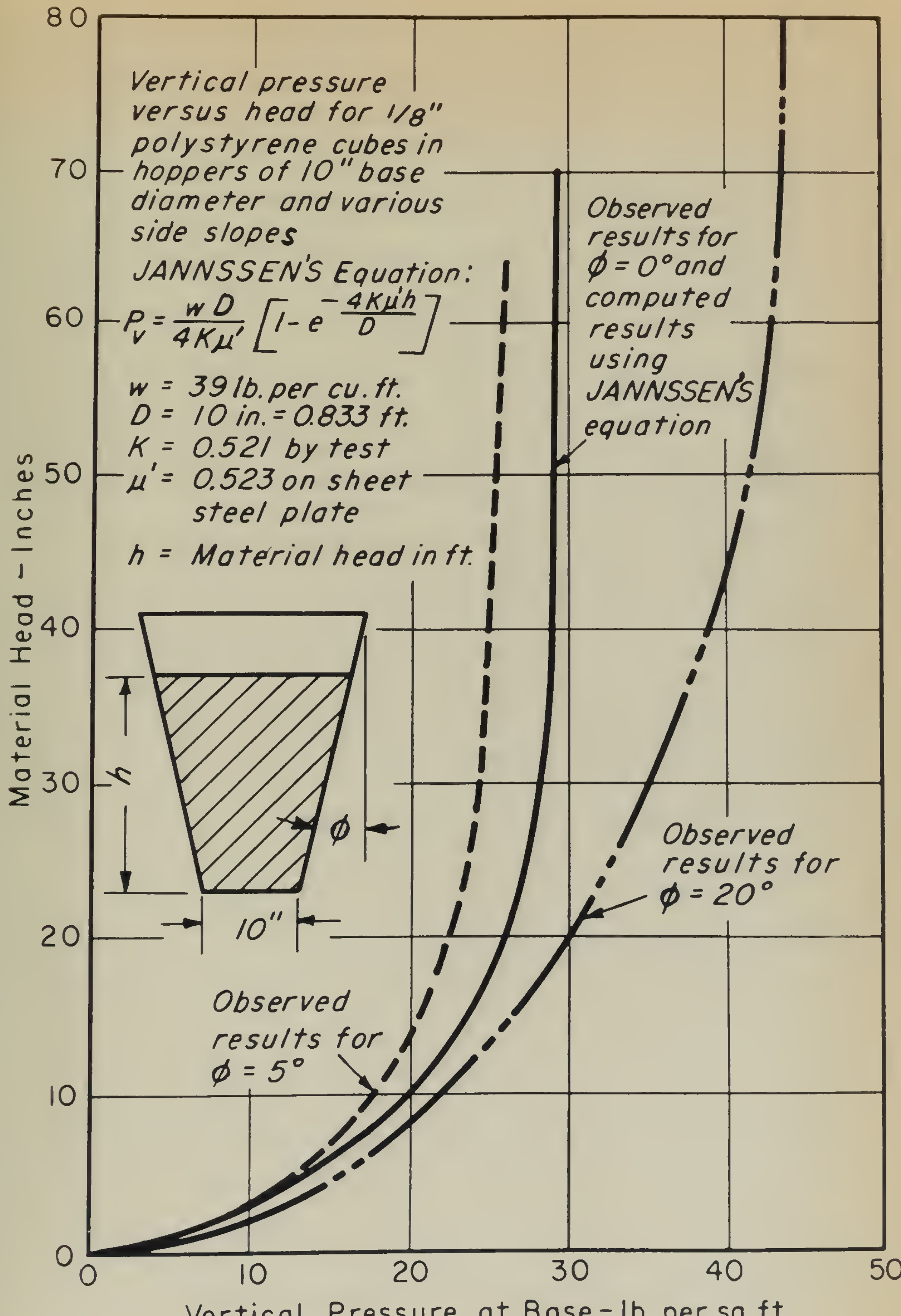

Fig. 11

Verification of Janssen's equation

(Courtesy of Richardson Scale Company)

and at present work is being conducted to develop such a formula.

In order to compare the empirical data with the theoretical it was necessary to determine the value of the ratio K for the system used. Pliessner[22] found the ratio of the lateral to the vertical pressures varies with the material depth, the type material and the bin. Rudd found that K varies with the height of material in the hopper and the diameter of the hopper. To measure this value a unique device was designed called the "K-determinator". This mechanism consists of a pressure tank containing a flexible thin-walled plastic cylinder, the ends of which are rigid. The specimen is placed in the cylinder which is then installed in the tank. "A given pressure is then established inside the tank, creating a known lateral pressure on the sample. A vertical pressure then is applied by a piston on top of the cylindrical sample. Vertical pressure is increased until the rubber walls start to give way. At this point, the vertical and lateral components have been determined. Thus, Pl/Pv equals K."[23] The cylinder was constructed to conform to the height-to-diameter ratio of the hopper used in the program.

3. Nature of Flow

Thus far we have become familiar with the laws of semi-fluids and the factors governing the action of bulk materials in bins. These laws explain the loading and force distribution that occurs under static conditions. Let us now consider how bulk ma-

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22. Ibid 1, page 349

23. Ibid 12.

terials flow from bins. To do this, it is necessary to qualify the types of flow. One might consider as an analogy the viscosity property of fluids for in semi-fluids, flow ranges from free-running flow to what has been called plug flow. The transition state is not readily defined but it is conceivable that a flowability classification could be developed. While observing the flow of material in various degrees of confinement Wolf and von Hohenleiten[24] determined that there were the two major classifications of flow as mentioned above. Under certain conditions the transitory type of flow occurred as stated in the following:

> "At least three types of flow have been observed in these experiments .... Granular flow, in which there is distinct movement of one particle with respect to the others, occurs with fairly dry coal. Plug flow, in which the whole column of coal moves as one mass with no apparent relative movement of individual particles, occurs at high moisture contents. With moisture contents corresponding to the trough of the "density-moisture curve," there exists a transition type of flow which resembles viscous flow. The transition from granular flow to plug flow cannot be clearly defined and is influenced by the physical composition of coal. In this region of change in type of flow, it was difficult to obtain reproducibility in many tests."

In order to better understand how these systems of flow perform, let us study the two extreme conditions individually.

<u>Streamline Flow:</u> A cylindrical bin has been filled with a free flowing material and the discharge gate has just opened. The first granules to move out are those immediately above the discharge

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24. Wolf, E. F., and H. L. von Hohenleiten, "Experimental Study of the Flow of Coal in Chutes at Riverside Generating Station," <u>Transactions of the A.S.M.E.</u>, October 1945.

area. There is a general settling of the entire column of material above the discharge on up to the free surface. As the free surface subsides the void is filled with the adjacent free surface particles. This funneling continues from the free surface developing a conical surface of flow, the apex being the discharge area. During this movement there is a lining of static material present between the wall and a conical surface concentric with the flow cone. This surface, however, has an angle approximately the angle of rupture. The material in this area does not commence to flow until all other material has left the bins. The volume referred to is part of the first material to enter the bin upon loading, and, as we have seen, is the last to leave. For materials which are subjected to degradation, such a pattern presents a stowage problem. As is often the case the bin is replenished before being voided. This results in retention of the material in the static region over an extended period of time.

Sandstrom[25] reports on what might be considered a birds-eye view of the flow of material. In his experience he had the opportunity to make daily observations from catwalks of the flow of coal from some ten bins. He described these observations as follows:

> "Starting off with a surcharge there was a general subsidence for about two feet. Then for another two feet there was a leveling of the surcharge, which indicated a piping, or funnelling, down the center of the mass of coal. In the next foot the surface became dished, the dish becoming a crater by the time the rim of coal was at mid-height of the wall of the bin. From there on the crater was an envelope of a cone, an element of which was inclined at an angle considerably greater than the angle at which the coal

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25. Sandstrom, C. O., "Building Bins of Wood and Steel," <u>Chemical and Metallurgical Engineering</u>, Vol. 47, 1940, pp. 22-25.

> would slide down a flat plate; which greater angle was obviously due to the circumferential restraint. Shortly before the discharge opening was exposed, the coal began to flow down the surface of the envelope, which continued until a break in the surface occurred, exposing the steel hopper. This relieved the circumferential restraint, and the coal began to slide down the hopper, continuing until the hopper was empty."

Similar experiences as Sandstrom's have been reported by others but the most effective description of the flow of materials came out of the work conducted by Rudd[26]. As has often been said, one picture is worth 10,000 words and in this case that certainly is true. Rudd took photographs of the flow of materials in a glass-fronted bin. In this particular case the flow represented that of the center cross-section of a bin. Colored polystyrene granules were stratified to dramatize the results. (Reference Figure (12)). The flow pattern may be observed in Figures (13) and (14). Rudd commented on this pattern as follows:

> "A central column of flow exists above the discharge opening and is as wide as the opening. If material was being put into the bin at the same rate as it was being withdrawn, material outside the area of the central column consequently would remain static. Material flows into the central column area as soon as the central column is discharged."

Thus it may be seen that stream line flow follows a pattern of columnar discharge, the column developing into an inverted cone as the material is removed. One may design a self-cleaning bin by using slope angles greater than the angle of repose. This does not change the first in- last out status of the static material previously defined.

In general usage bins are not fully discharged before

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26. Ibid.

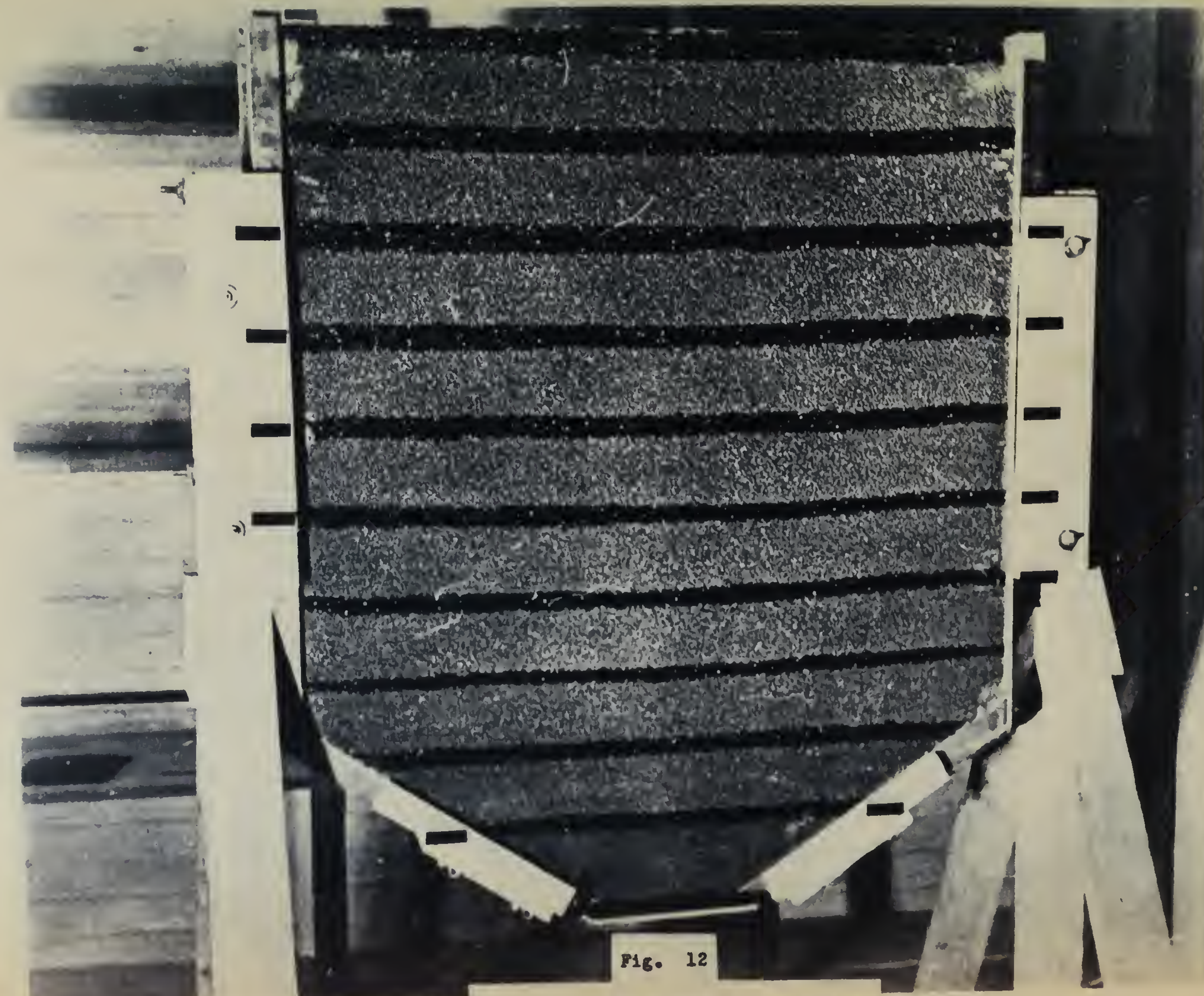

Fig. 12

Stratified Polystyrene in Model Bin

(Courtesy of Richardson Scale Company)

Fig. 13

Streamline Flow Pattern - Discharge Commencing

(Courtesy of Richardson Scale Company)

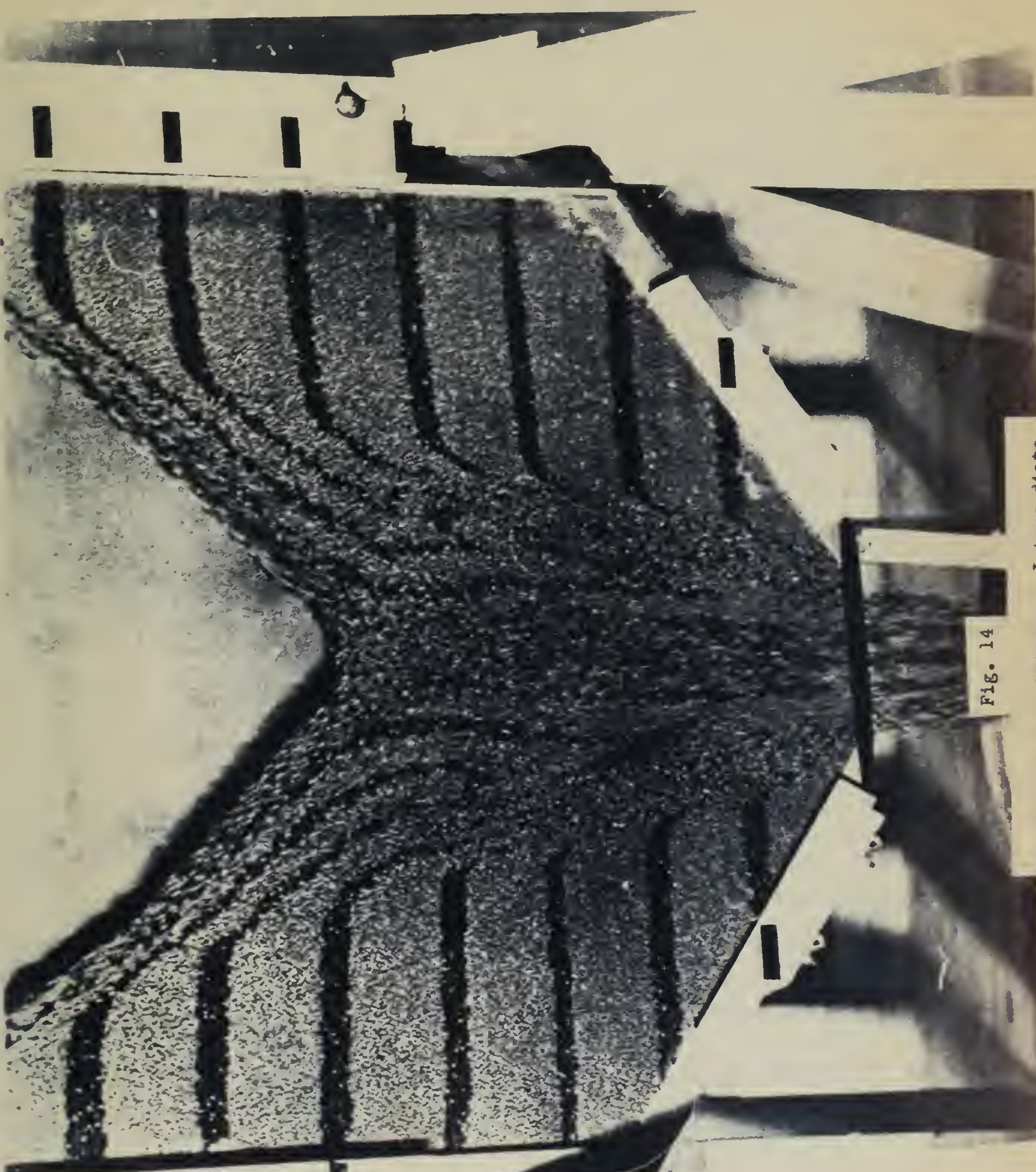

Fig. 14

Streamline Flow Pattern - Intermediate

(Courtesy of Richardson Scale Company)

replenishment but are replenished with use. This presents the problem that the material which remains in the static area may be subjected to an extended period of storage encouraging fermentation and infestation. In systems where such is the case this problem must be recognized and dealt with accordingly. One rather expensive solution is to have an auxiliary bin into which the material may be rotated thus re-orienting the mass.

Cohesive Flow: Operating personnel have defined two common occurrences in handling cohesive materials,-- arching and ratholing. Arching is the suspension of the mass above some elevation within the bin. The material has transmitted its weight to the side walls of the bins by springing an arch across the bin. It will remain in this state of unstable equilibrium until physically disturbed. Ratholing is the flow of material through some random channel much like the path of vermin in a mound. This occurs when the force of gravity of a given mass of material is sufficient to overcome the shear strength of the material. When the shear mass is of the volume immediately above the discharge area this is referred to as plug flow. Ratholing developing into arching in a deep bin is highly conceivable. Conversely, ratholing could also result from continuous random spalling of arches. It is not intended to segregate these two items. At present, however, there are two definitions of the forces involved in cohesive flow. One best defines arching while the other is more compatible to ratholing.

Of the many attempts to define arching and to rationalize

its occurrences, the author feels that Lee's[27] description is the most effective:

> "It can be stated that granular materials move downward towards the hopper outlet in streamline flow. But that non-granular and cohesive materials move by the continual collapsing of arches at various heights above the outlet .... An arch may be considered as a curved system of pressure forces springing from one supporting surface to the other. This definition is convenient in explaining the way granular materials - such as grain - act in deep bins; and also in explaining the slight reduction in pressure actually observed with granular materials at the foot of a vertical side in a shallow bin. It would be natural to assume that arch forces intersect the wall at 90 deg., but this is not so. Assume material flowing downward between two parallel walls. There is pressure against the walls and a frictional drag. For a unit height a triangle of forces may be drawn as shown in Figure (15). The lateral forces, it is evident, must be inclined at the wall but are horizontal in the center. Thus we have an arched system of pressure forces between two parallel walls. It is not so obvious that the lateral forces at the wall are inclined (at the same angle) even under static conditions, i.e., with materials at rest. However, the well-known Janssen analysis gives results closely in accord with experimental findings.
>
> An arch may be more conventionally defined as the structure containing the system or compressive forces. In the case of granular materials the arch is not self-supporting but rests upon material beneath. With that understanding, this definition is quite consistent with the Janssen analysis. This second definition has obvious application to cohesive materials. Assume an irregular arch left after material beneath has been discharged. There are two destructive tendencies: Material hanging from arch may fall, pulling material with it out of the arch proper; material in the arch is under compression and there is a spalling tendency on the lower side of the arch. If the span is too great, the arch collapses and material is discharged. On the other hand, arching with a high degree of curvature is inherently stronger. Compressive forces on the under side are less, and so is the spalling tendency.

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27. "Design of Hoppers for Use", by Chesman A. Lee, Chemical Engineering, May, 1953.

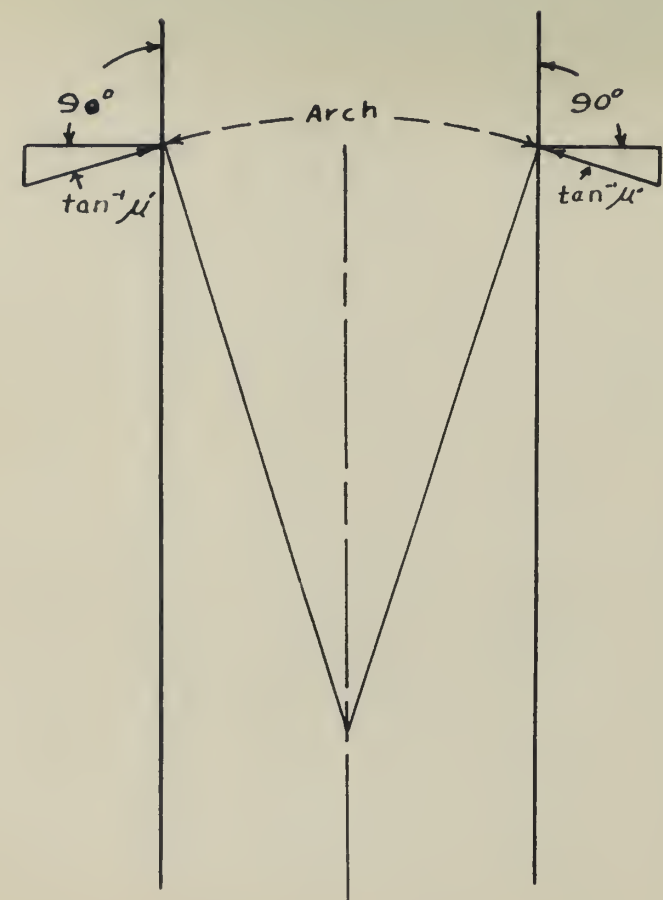

Fig. 15.

Arch System of Pressure Forces

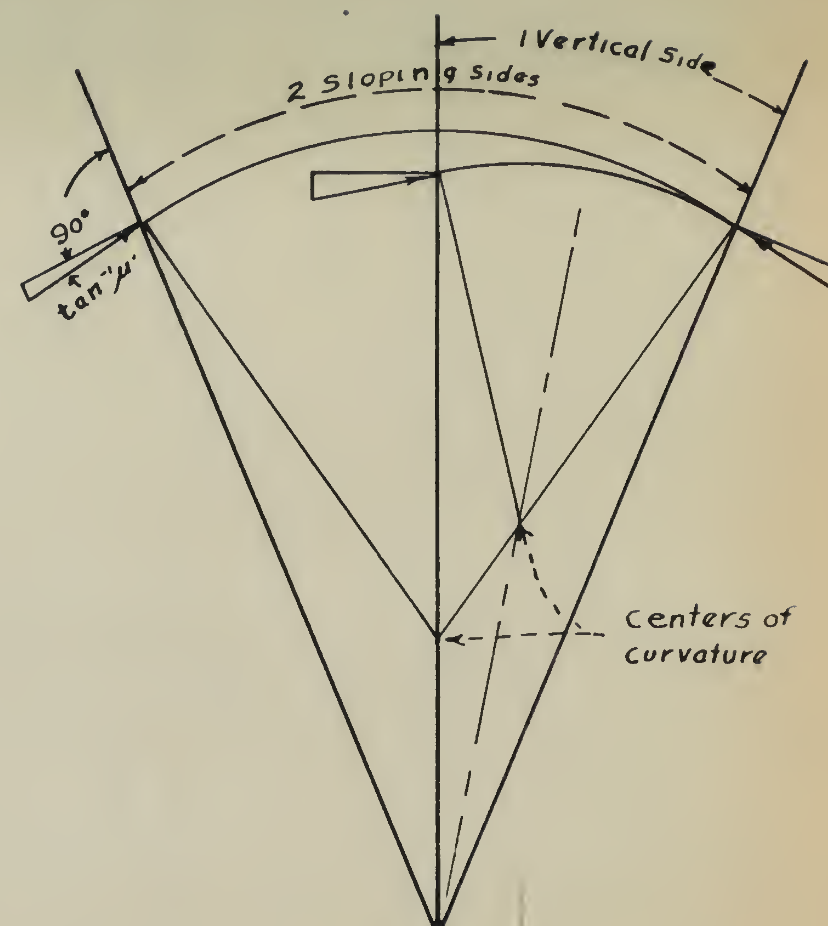

Fig. 16.

Vertical Side Arch Versus Sloped Side Arch

(Courtesy of Chemical Engineering)

We have already seen that a vertical side in a hopper speeds up the discharge of granular materials, but is a hindrance with cohesive materials. We are now ready to explain and justify these statements. In Figure 16 the compressive forces in an arch intersect the wall at the angle 90 degrees + $\tan^{-1} \mu'$. Here μ' is the coefficient of sliding friction of material against wall. The center of curvature is located easily as the intersection of the normals drawn to the force resultants at the walls. In Figure 16 two diagrams have been combined. One is for a symmetrical hopper; the other shows the effect of making one side vertical. Obviously the arch is inherently stronger (shorter radius of curvature) with a vertical side. Experiment shows that a vertical side reduces the pressure on the bottom even with granular materials. More of the weight is carried by friction, that is, by the walls themselves."

It should be mentioned at this time that Lee's[28] interpretation of the value of a straight side used in conjunction with a sloped side is a minority opinion. Many practitioners laud the advantage of straight sides wherever possible. These statements are usually made in view of the individual's experience; however, none have attempted an analytical approach. (More will be said about this under Flow-Factor).

Bulk materials have a shear strength, hence, to induce flow it is necessary to set up a stress to overcome same. This is what occurs in a bin when the discharge gate is opened. The force of gravity acting on the material immediately above the discharge is sufficient to overcome the shear strength. It is entirely possible that the discharge area is insufficient to permit flow. In this case it is necessary to resort to lances and vibrators to provide

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28. Idem.

the additional stress and agitation required. After the plug has moved out, the force network, compressive and cohesive, is sufficient to retain the rest of the material in the bin. Further agitation is needed to cause this mass to flow. The nature of plug flow is aptly demonstrated by Figures (17) and (18). These are additional pictures recording the flow of material in bins as taken by Rudd[29]. The plug of material over the discharge moved out, leaving a rathole. Some break-away occurred as a result of the inertia of the plug.

4. Design Factors

From the preceding coverage one might surmise that bins and hoppers are not used or are going out of use. Such is not the case, for of the many varied materials stored in bins relatively few demonstrate extreme cohesive features. Further, as is true with most engineering, there are compromises which may be made. In this section will be presented those design factors which are determinants in flow. Suggestions to aid flow culled from the experience of others will be presented also.

Slope of Wall: The wall angle referred to is the slope of the hopper section of the bin. The most common shape of this section has the sides sloping toward the discharge at equal angles. The main advantage of such a shape is that a given volume of material is stored with a minimum head room. Hardinge[30] and Sandstrom[31] are in favor of a bin whose hopper section has one or two vertical sides. The vertical

29. Ibid 12.
30. Hardinge, H., "Bin Shapes and Feeders," Industrial and Engineering Chemistry, Vol. 27, 1935, pp. 1338-1341.
31. Ibid 9.

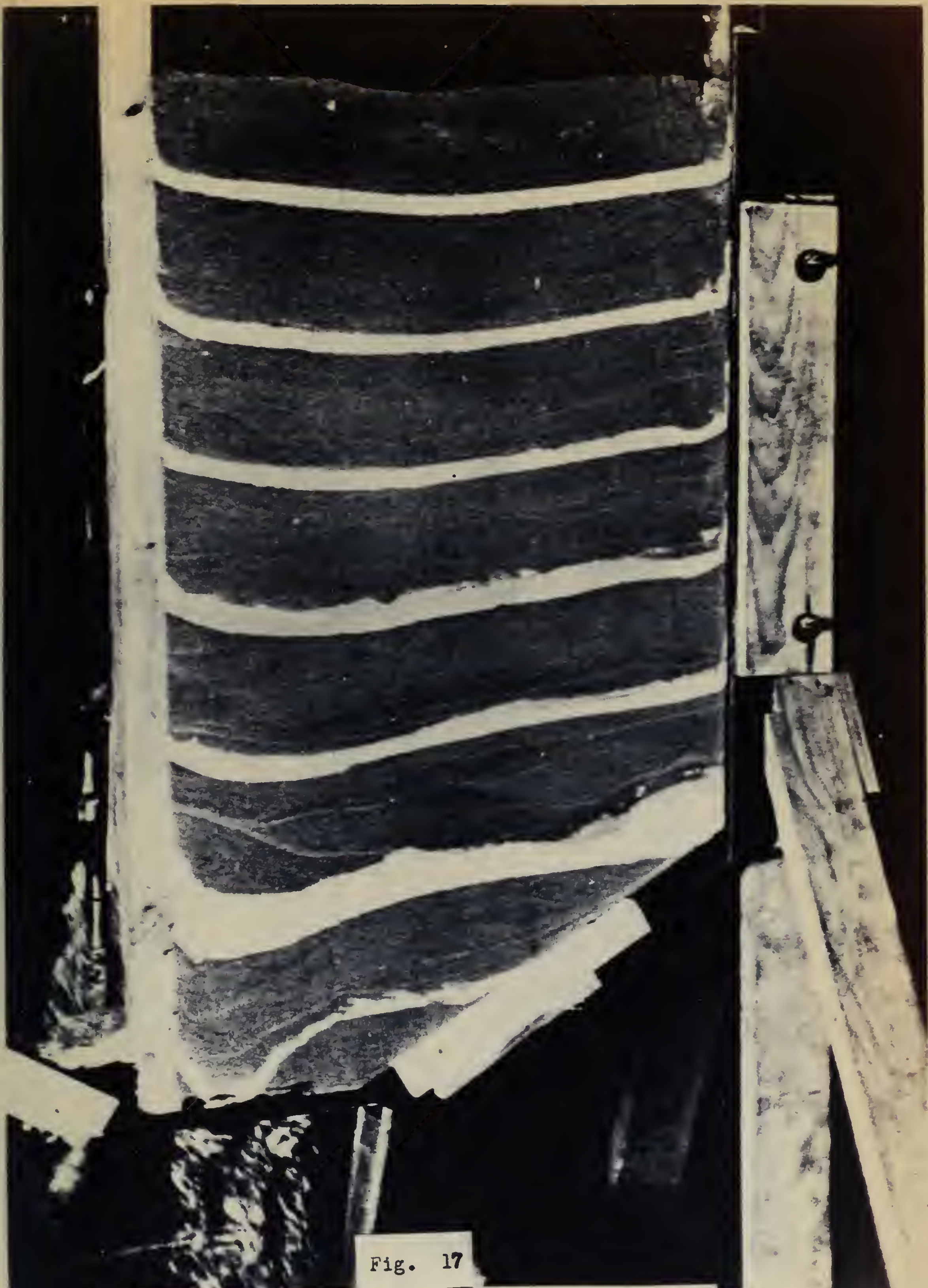

Fig. 17

Stratified Cohesive Material - Discharge Commencing

(Courtesy of Richardson Scale Company)

Fig. 18

Stratified Cohesive Material - Discharge Ceased

(Courtesy of Richardson Scale Company)

sides tend to keep the material flowing down and thus prevent springing an arch. Hardinge further recommends a bin having vertical sides with no hopper section but which has multiple discharges as shown in Figure (19). Such a design practically precludes bridging and minimizes segregation. The major difficulty with this type of bin is the feed must be regulated from each discharge. Lee[32], however, does not feel that a hopper section with a vertical side is of value when handling cohesive material. It is his feeling that a vertical and slope side combination encourages bridging and actually makes for a stronger bridge. His analysis of this condition was presented earlier under the discussion of arching. On this point we have a difference of opinions among people with extensive background in this work. In correspondence which the author had with Rudd, he took exception with Lee's conclusion and stated that in his experience straight bin sides are of considerable help. This difference of opinion refers to the usage of a combined vertical and sloped sided hopper for cohesive materials. There is no such argument about the value of vertical sides for streamline flow. For free flow materials it is recommended that the sloped sides be steeper than the angle of

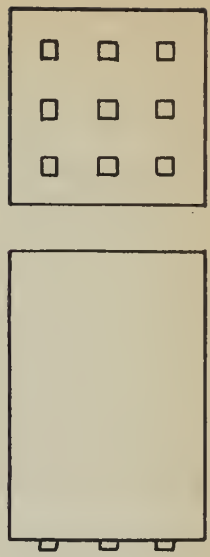

Fig. 19

Vertical Side Bin with Multiple Discharge

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32. Lee, Chesman A., "Design of Hoppers for Use", Chemical Engineering, May 1953.

repose. This will make the bins self-cleaning. Where possible it is recommended also that vertical sides be used.

Discharge Area: There are two items to consider about the discharge area: its location and its size. For cohesive materials the discharge area may be the prerequisite to flow. That is to say, a minimum discharge area exists for every cohesive material below which flow is not self-induced. In accordance with the plug flow analogy this area is the base area of a column of sufficient mass to shear away from its surroundings. It may be possible in the future to determine the discharge area required to induce flow in a particular system. To do this, it is necessary to know the shear strength of the material and the stress set up in the system. Stress is a function of the system. Stress is a function of the characteristics of the material and of the design of the bin. Rudd[33] feels that his present investigation will lead to an equation capable of reliably determining this stress. In addition to the foregoing application, we must not overlook the fact that the flow rate of the material is a function of the discharge area.

In the past it was quite common to locate the discharge gate on the bottom of a sidewall. This practice was generally discontinued when Jamieson[34] demonstrated that although the pressure decreased on the gate side it increased two to four times the lateral pressure on the opposite side. Hence, the discharge is now commonly located in the center. Proponents of a vertical side wall require the discharge to be located off-center.

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33. Ibid 12.

34. Ibid 1, page 334.

Chutes and Feeders: Between the bin storage and the point of usage materials are frequently transported through drop chutes or feeders. The material reacts within these chutes the same as in the bin. The tendency for flow stoppage is more acute in chutes due to the reduction in cross-sectional area, the radical changes in flow direction and the presence of internal obstructions. The three items mentioned have plagued the industry for years. The prime purpose of the experiments conducted by Wolf and von Hohenleiten[35] was to define the factors most responsible for flow stoppage in chutes and to develop methods to improve flow. They concluded that other than the physical properties of the material the three factors listed were the major contributants to flow stoppage in chutes. On one occasion, they found that such a minute obstruction as a burred edge was sufficient to raise the angle of slide to 90 degrees. It is,therefore,advisable to keep the internal surface free of obstructions such as a raised nut head, a turned pipe edge, raised flange joint or a raised circumferential seam. As a result of their work, Wolf and von Hohenleiten[36] found that for most efficient usage of chutes the following design features should be maintained:

"1. Avoidance of sudden constrictions and sharp changes in direction.

2. Minimum angles of convergence of lines of flow, preferably approaching zero.

3. Minimum practical taper in hoppers.

4. Maximum possible angle of inclination with horizontal throughout the system.

5. Use of round shapes in preference to square or rectangular shapes."

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35. Ibid 3.

36. Idem.

For feeder lines where there is a transition from one chute configuration to another, or where there is a very sharp change in direction the use of "breakaway" is recommended. A "breakaway" is an enlargement around the transition point which allows the material to fall out in all directions as is shown in Figure (20). This alleviates the compaction which would occur with normal transition.

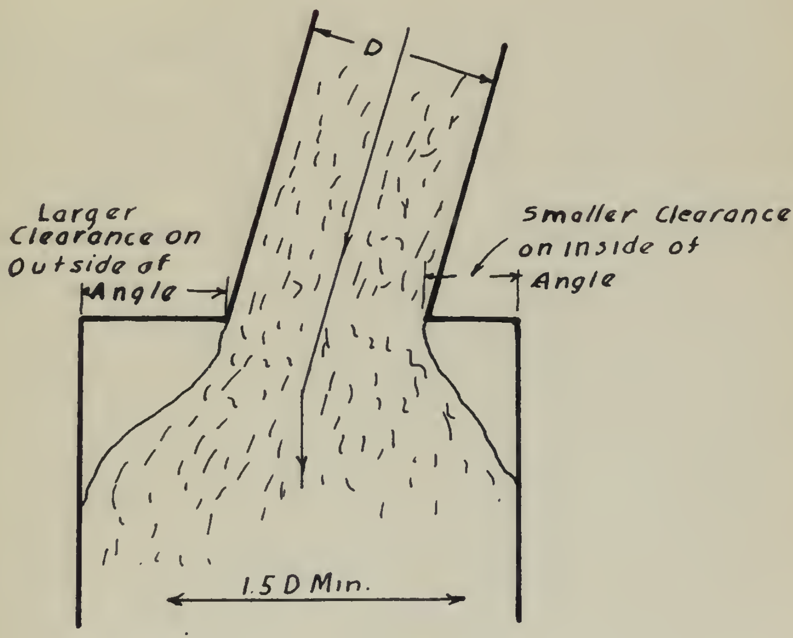

Fig. 20

Schematic Sketch of "Breakaway"
(From Transactions of A.S.M.E., October, 1945)

One must use conical shaped chutes with caution for, as determined by Wolf and von Hohenleiten,[37] "while permitting free flow as an individual unit, (they) show a tendency to compact the coal by their powerful wedge action when they become part of a system and whenever

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37. Idem.

the ratio of their lengths to smallest diameter exceeds approximately two." "On the other hand, short cones of moderate slope work fairly well so long as the ratio of their lengths to smallest diameter does not exceed 1.5 to 2. Concentric cones have an advantage where it is desired to achieve a reduction in cross-section while eccentric cones can be employed effectively where both lateral transition and reduction in area are required."

Physical Appendages: The severity of the problem of flow stoppage is evidenced by the many external and internal contrivances designed to maintain flow. Some have had a degree of success while the majority have only added to the frustration of the moment by their incompetence. The sledge hammer has ruptured many a bin and is still a standby "tool". A refinement of the sledge hammer is the mechanical vibrator, both high and low frequency. This unit is fairly effective when used on bins holding flours or light weight material. A common error, however, is its usage prior to opening the discharge which only results in added compaction. To envision the size of some of these mechanical giants, call to mind that an average bin might have a 14-foot square cross-section and stand from 35 ft. to 45 ft. high. Hudson[38] has recorded the use of links of chain suspended in an arc from the top to the bottom of a bin. The chain, being flexible, would give in a high pressure area and agitate material along other sections. When the material left the bin the chain would slack off, flexing elsewhere. Hudson also had observed the use of 24-gage stainless steel

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38. Hudson, W. G., "Bins, Bunkers and Silos," Power Plant Engineering, Vol. 50, May, 1946.

sheets suspended in the bin. This was reported as being effective in combating bridging.

A sprocket-driven chain was introduced by Sandstrom[39] as one of the many false attempts. An endless chain is suspended from a sprocket in the center of the bin. The sprocket is supposed to pull the chain through the material and induce flow. The downward moving chain must move with the same speed as the upward moving section or the chain will leave the sprocket. This would not occur in anything less than a fluid material. In semi-fluids, the downward chain, - being subjected to gravity only, would have the same status as the man who stands knee-deep in soybeans. In plug flow one would observe the chain merrily rotating in the void that the plug once occupied. Sandstrom[40] presents a series of stirrers and agitators, one similar to a milk shake mixer while another resembles a Waring Blender. All such devices could only be used with fine materials. If one would consider that bridging occurs above a screw conveyor then it is not difficult to conceive how the same would occur here.

For use in small, shallow bins Hardinge[41] enumerates several belt pulley systems which pass around the inside walls. He is particular to state, however, that such belt pulley systems are restricted to small systems due to the excessive power required to overcome the friction on the belt. There have been many other devices designed to be

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39. Ibid 9.

40. Idem.

41. Ibid 10.

suspended or attached some way within the bin. Usually these attachments are immediately over the discharge area. The driven devices all require a great deal of power to operate. Whenever any of the internal devices are brought to mind they are accompanied with this associated thought, "When arching occurs several thousand pounds of material may be suspended, all or part of which will plunge when flow is restored. This dynamic force will be directed at these internal appendages. Are they capable of withstanding this shock?"

A different approach to this problem is embodied in an item developed by Tainton[42]. This method utilizes an inflatable neoprene panel which is affixed to the inside wall of the bin. The panel or panels are strategically located in known trouble areas. The neoprene panel is inflated using low pressure air to a maximum amplitude of 18 inches. The action of the panel is to displace the arched section so that upon deflation the material is unsupported and commences to flow. In its completely deflated condition the neoprene panel hugs the side walls of the bin. It does not decrease the volume nor is it a "protruding thumb". (See Figure 21). In Figure (22) we may see how the neoprene panels function to maintain flow. In (a) we see how an arch is displaced into the discharge area. In (b) the funnel is destroyed and the material again moved over the discharge. The above described item is a commercial item. The author wishes to state that it was given such extensive coverage here because to the best of his knowledge it is the only product of

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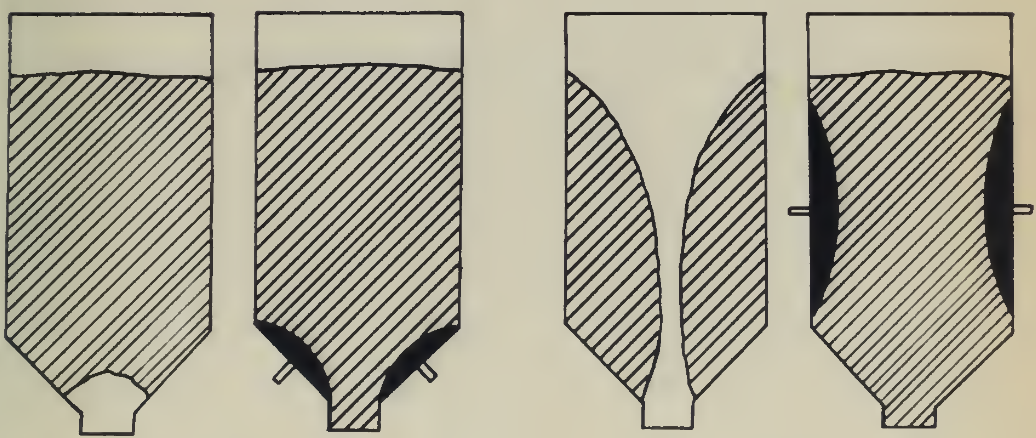
42. Davis, E. A., and Rolfe Pottberg, "Problems in Storing and Handling Pulverized Materials," Mechanical Engineering, March 1952, Vol. 74, pp. 246-248.

Fig. 21 Installation of Inflatable Neoprene Panels
Courtesy of Geroter May Corp., Balto.,Md.



(a) Panel Action in Hopper Section

(b) Panel Action in Vertical Section

Fig. 22 Schematic of Neoprene Panel Displacement
Courtesy of Geroter May Corp., Balto., Md.

its type. No endorsement or certification is intended.

Still another method of preventing arching is the use of aerators. These are units capable of injecting air into the bin through various types of filter pads. They are used primarily on bins containing light-weight materials. The injected air serves as an agitator, a dehumidifier, or in some cases as a media of transportation. When used for transporting material the injected air agitates and carries in suspension the particles of the material. Usually in such instances a vacuum draw-off is also utilized. To define a particular system in detail would be defining a producer. There are several manufacturers of aeration systems, all of which are ready to provide details of their particular system. It may be said that aeration units and air-flow units provide a very effective method of handling the type materials for which designed. The units are attached to the outside of the bin with filter pad access to the inside. They are also readily adapted to chutes and feeders.

5. Quantitative Analysis: Flow Factor

Flow-factor is the term introduced by Jenike[44] to define the relationship between the compressive strength of a material under compacting pressure and the flowability of that material. Jenike has developed a series of formulas which will determine the physical characteristics of bin design to provide flow. Before we look at these formulas, let us first consider the novel approach leading to

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44. Jenike, A. W., "Flow of Bulk Solids in Bins," Bulletin of the University of Utah, Vol. 45, No. 4, March, 1954.

their development. The conditions of flow stoppage are defined as doming and funneling; these have been previously referred to as arching and plug flow. Their occurrence is attributed to the design of the bin, the material used and the operation sequence of the bin. It is felt that the load-unload-refill sequence of material in a bin is a contributing factor to obstructions (stoppage).

Under confinement material has a certain strength capable of supporting an arch or dome across a limited span. To determine the maximum span possible it is necessary to define the maximum strength of the material. The strength of the material is a function of the pressure to which it is subjected. Jenike[45] defines three sources of pressure: "The weight of material, this is called static pressure, the impact of falling material, referred to as impact pressure, and vibration of the bin or vibrational pressure." Since pressures vary throughout the bin both with elevation and direction, the centerline pressure is chosen as representative. A family of curves representing the possible centerline pressures in a hopper are shown in Figure 23. These are not unlike the pressure distributions found by others. The ratio v/w is defined as the unit pressure. The analysis is concentrated on the hopper section of the bin because tests on models indicate the vertical portion of a bin had little effect on pressures in the hopper. Thus, the vertical section does not affect the flow of material through the hopper. The pressure distributions in the hopper results in a wedging which compacts the material laterally increasing

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45. Idem.

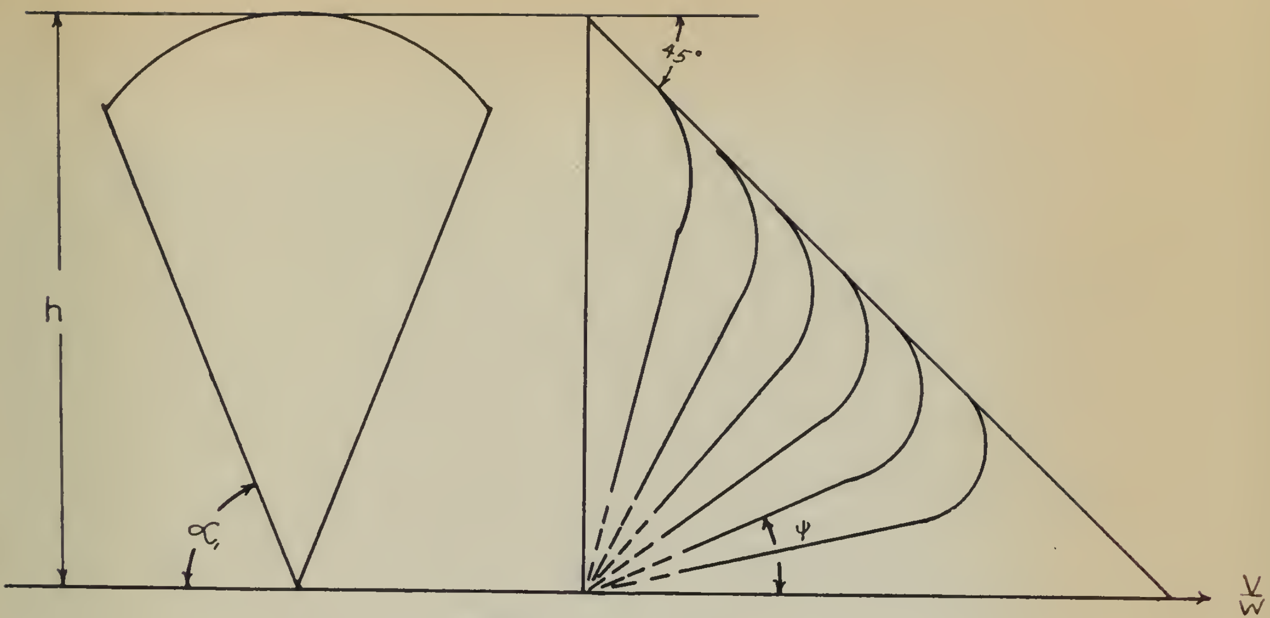

Fig. 23
Centerline Pressures in Hopper

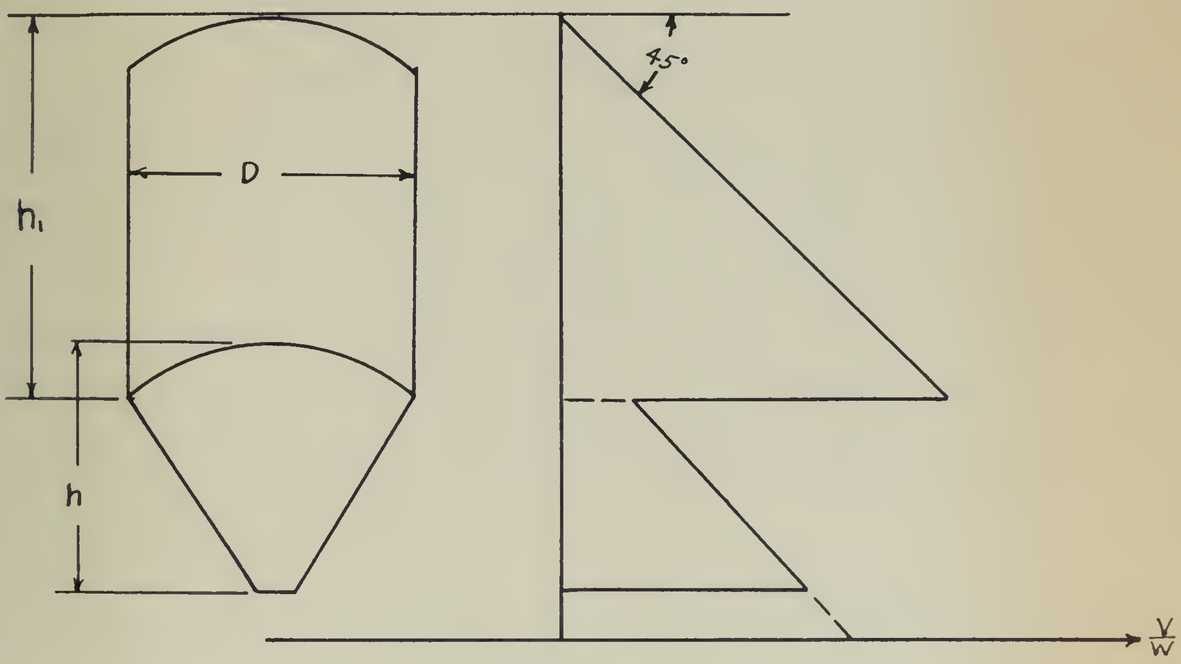

Fig. 24

Maximum Pressure Areas in a Bin

(Courtesy of Andrew W. Jenike)

the horizontal component producing an arching effect. At the same time the vertical centerline pressures decrease. The influence of the walls is greater the narrower the section of the hopper.

Maximum Pressure Areas: The result of experiments conducted by Jenike[46] indicate that there are two cross-sections of the bin at which pressure reaches a maximum: "One is at the opening of the hopper and the other at the transition from the vertical portion of the bin to the hopper." (See Figure (24)). "As long as h is larger than h_1, the critical condition for doming remains at the outlet of the hopper. For h_1 larger than h, however, the largest pressure occurs at the transition and it is there that material develops the greatest strength."

Pressure Breakers: Now that the major areas of potential pressure and thus material strength are pin-pointed, Jenike[47] suggests some internal appendages to relieve the pressure. It is proposed that a horizontal shelf be placed in the center of the hopper as shown in figure (25). This will retain the material and help reduce the pressure below the shelf. In tall bins, a vertical partition as in Figure (26) is proported to reduce pressures at the transitional area.

> "The principle of operation is based on the fact that the partitions reduce the effective diameter of the bin. Horizontal ledges within the bin can also be used to control the compacting pressure in a bin. (This is shown in Figure (27)). It is advisable to keep all peaks of pressure below the pressure at the opening of the hopper.

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46. Idem.

47. Idem.

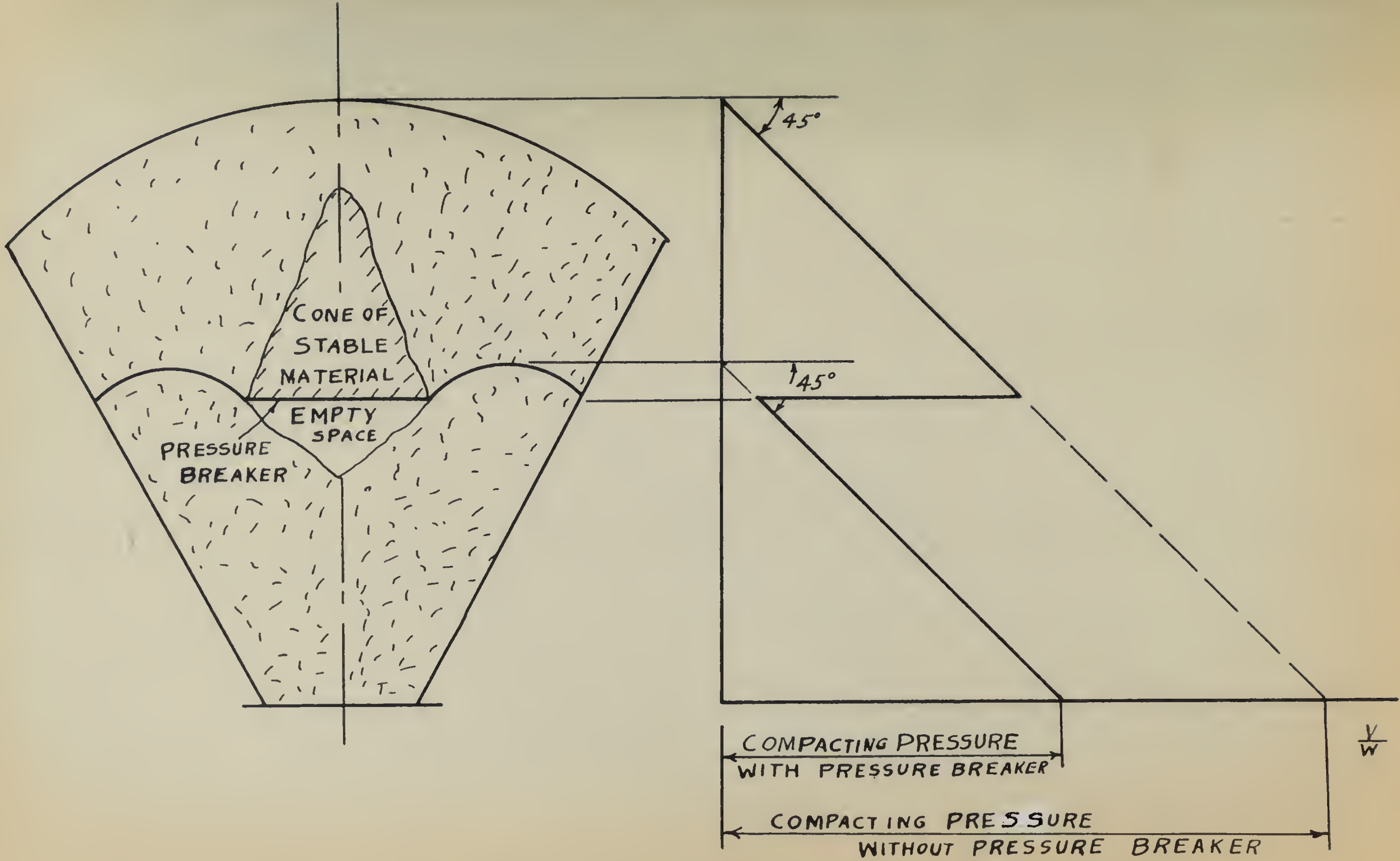

Fig. 25

Pressure Breaker Suspended Over Discharge Area

(Courtesy of Andrew W. Jenike)

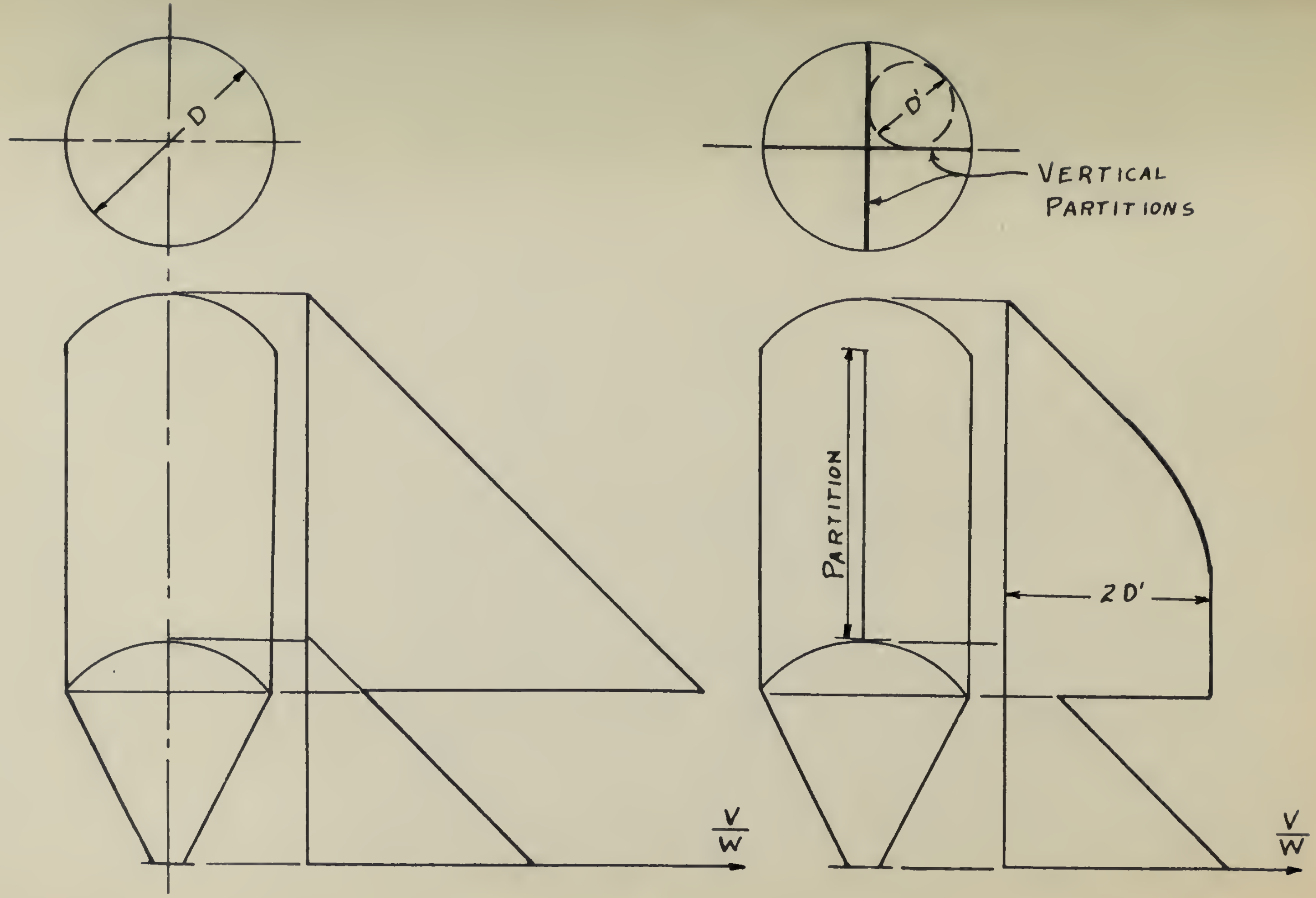

Fig. 26

Vertical Partitions to Reduce Effective Diameter of Bin

(Courtesy of Andrew W. Jenike)

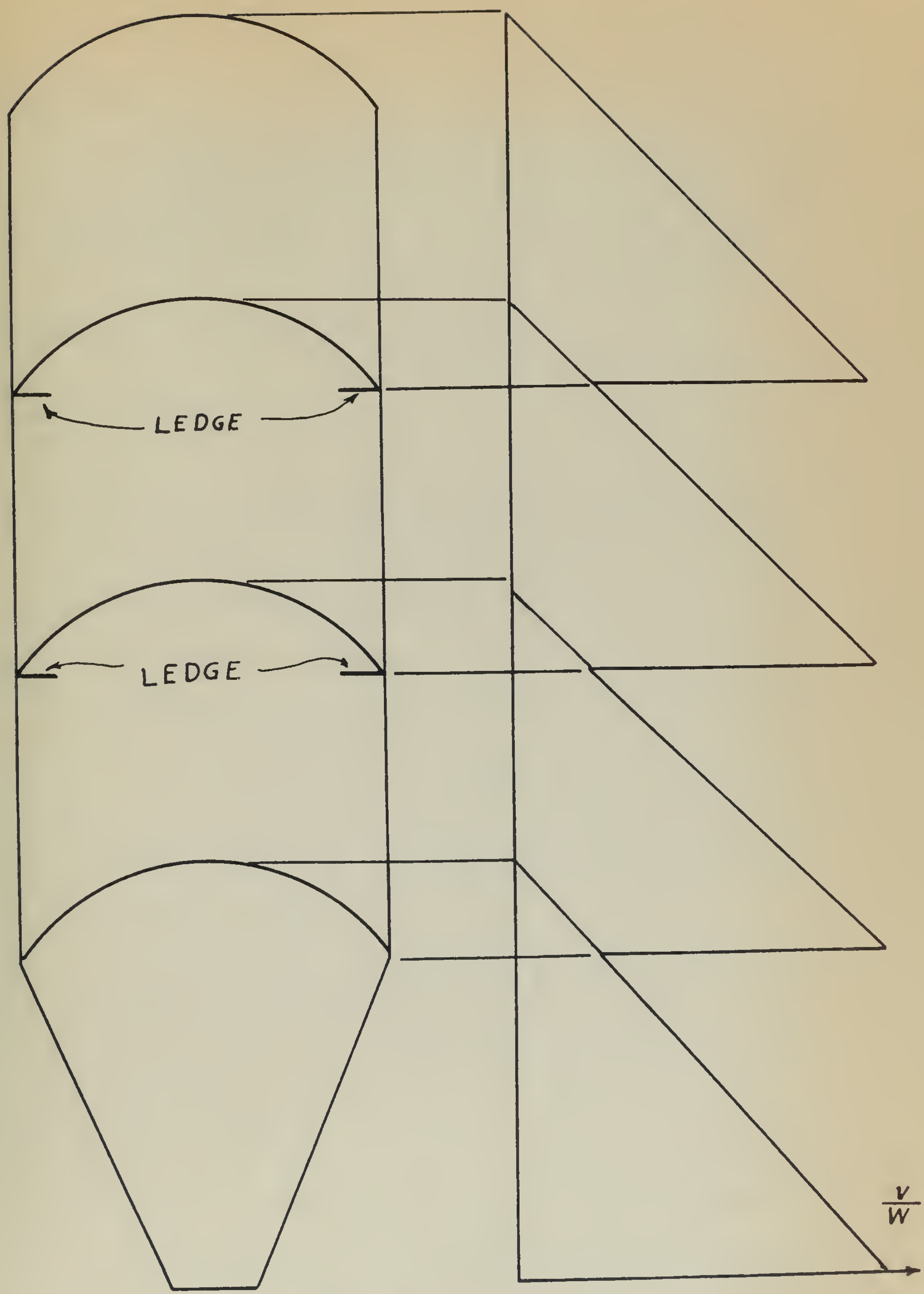

Fig. 27

Control of Compaction by Use of Horizontal Ledges

(Courtesy of Andrew W. Jenike)

In this way, the danger of doming across the funnel is eliminated." [48]

Flow Factor:

"The flow-factor is the ratio of peak compressive strength to unit weight of a material as a function of the ratio of compacting pressure to unit weight. Physically, the flow-factor is the radius of the largest circle over which the compacted material can form a stable dome." [49]

The flow-factor is expressed mathematically as

$$\frac{fc}{w} = F\left(\frac{v}{W}\right)$$

Where:

fc = unconfined compressive strength of material resulting from a compaction under pressure V.

v = largest principal pressure.

w = unit weight of material

$\frac{v}{W}$ = unit compacting pressure

The factor fc is a function of the cohesiveness of the material (c) and the internal friction (Ø) as demonstrated by the following:

$$fc = 2c \tan (45^{\circ} + \phi/2)$$

If we now assume that it is desired to prevent an arch across the hopper opening utilizing the flow-factor concept, the following formula may be applied.

$$B > b_h \; b_v \; \frac{fc}{w} \; \sin 2\theta_1$$

B is the length of the minor axis of the hopper opening. The coefficient b_h is governed by the horizontal cross-section of the

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49. Ibid 23.

opening, having values between one and two. The vertical element is the coefficient b_v. These coefficients introduce the physical aspects of cross-section, height and side slope angle. The angle θ_1 defines the anticipated intersection between the hopper wall and the dome surface.

To design a hopper opening capable of defeating the second type obstruction, funnel, one would apply the formula of flow:

$$D > B + d\,\frac{fc}{w}$$

D is the width of the bin. The coefficient d is dependent upon the ratio of the hopper opening length to width and also the ratio of the bin width to the hopper opening width.

The above formulas were developed and reported on in a previous paper by Jenike[50] entitled, "The Flow of Bulk Solids in Bins." A condensation is not attempted in this presentation as it is not felt that the mathematical analysis may be appropriately condensed. The reader is referred to this very enlightening treatise for more detailed information.

The empirical coefficients were determined and substantiated for a limited number of bin configurations and materials.

<u>Vertical Walls versus Sloped Walls</u>: There has been a difference of opinions in previously reported papers on the influence of vertical walls in a hopper. Here is an excellent analysis of the effect of vertical sides. A comparison is made between a hopper with two

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50. Ibid 44.

vertical sides and a hopper with inclined sides. The two hoppers are equivalent flow wise. Referring to Figure (28), the inclined hopper is represented by the broken lines. The following assumptions are made:

$$\alpha_1 = 60^\circ, \quad \phi^1 = 10^\circ \text{ and } m = 0.2.$$

m is the ratio fa/fc, a functional ratio of adhesion to cohesion. It is not appropriate to reproduce the graph from which the angle θ_1 is extracted, therefore we will accept at this time that $\sin 2\theta_1 = 1$ for the sloped wall and $\sin 2\theta = 0.5$. Applying the formula for the width $B = b_h b_v \frac{fc}{w} \sin 2\theta$, we have for the hopper with vertical sides:

$$B' = b_h b_v \frac{fc}{w} \frac{1 + 0.5}{2} = 0.75\, b_h b_v \frac{fc}{w}$$

For the inclined wall hopper,

$$B = b_h b_v \frac{fc}{w},$$

Therefore, $B' = 0.75\, B$.

From this analysis, Jenike[51] draws the following conclusion:

"As far as the length L' is concerned, it is evident that the equivalent length L will be somewhat greater than L'. This affects coefficient b_h, but we also note that a slight change of ratio L/B is of no practical significance in this case. The influence of vertical walls on funneling can be similarly estimated. In the planned view in Figure 28 a dashed ellipse represents the opening of the funnel likely to take place in the hopper with all walls sloping. The shape of the opening of a likely funnel in the bin with two vertical walls is also shown with a continuous line. The critical hoop pressure acts along this line. Therefore,

51. Ibid 44

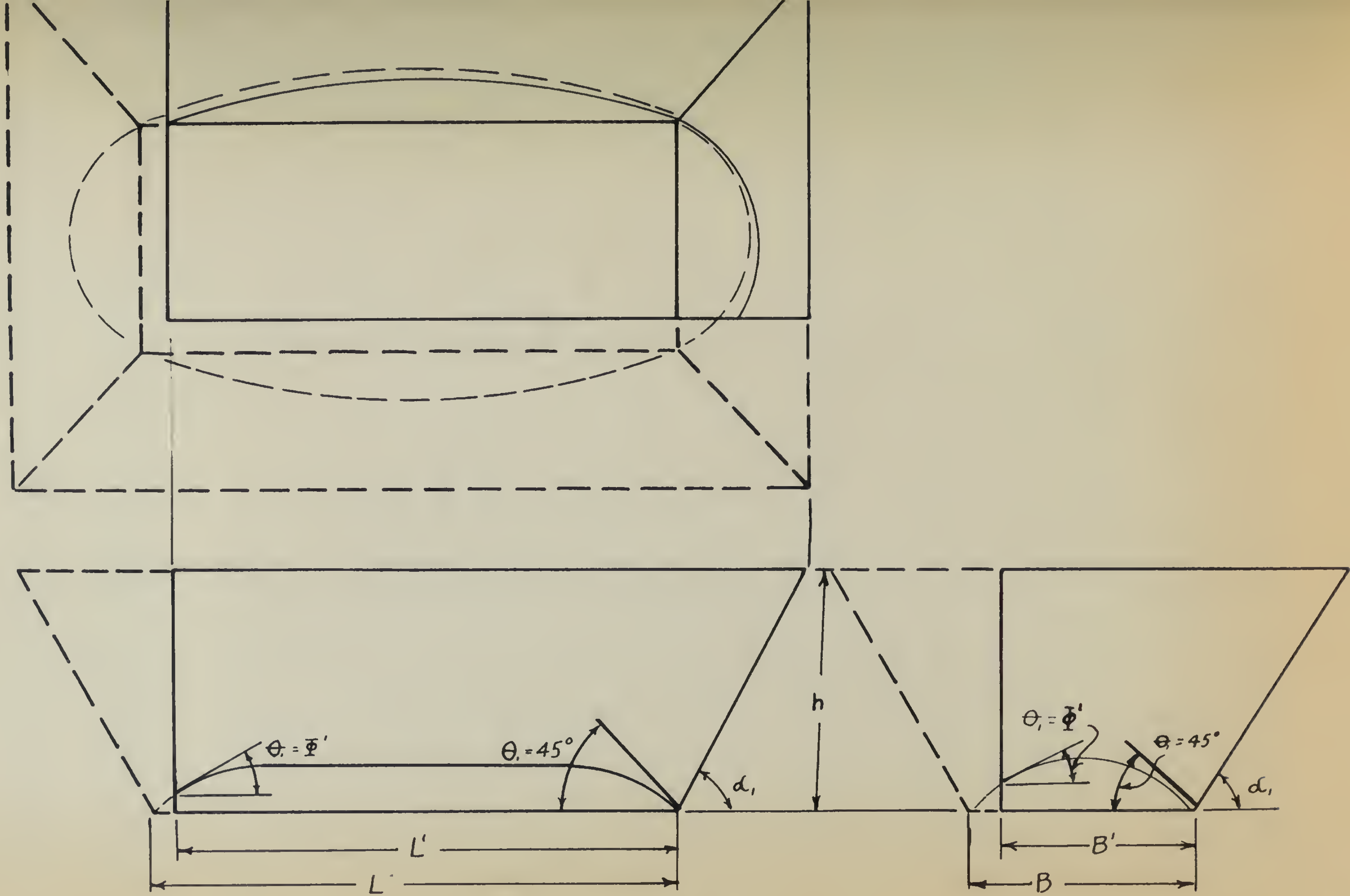

Fig. 28

Comparison of Vertical Sided Hopper Design versus Sloped Sided Hopper Design for Equivalent Flowability.

(Courtesy of Andrew W. Jenike)

at the points of intersection with the vertical walls, this line must lie within the effective angle of friction Φ' as measured from the normal to the wall. Comparing the shapes of the two funnels, we see that they differ very little and, so, vertical walls will have little effect on funneling."

6. Rheological Concept

The coverage of the literature would not be complete without mentioning the attempt to correlate rheological theories with semi-fluid flow. Silver[52] suggests that perhaps the principles of thixotropic change may apply to the behavior of semi-fluids such as grain, coal, meals, etc. Thixotropic materials are those materials which display a change in yield point with time. The concept usually applies to materials under stress. There were no concrete suggestions as to how this theory would apply or what the results of such an attempt might be. This was accompanied, however, by a general call to engineers to express themselves on the subject. Davis and Pottberg[53] rallied to the call with a very enlightening classification of material according to rheological concepts. The materials were arranged according to their physical state, i.e., solids, solid and liquid and solid and gas. Following this classification, the theory of flow for the particular phase was developed together with the anticipated response of the materials to three discharge devices. The devices are vibrators, aerators, and displacement panels. An example of their approach together with further information on thixotropic materials follows:

"Type C - Thixotropic Material: Very small particles with liquid in excess of that required to fill the voids such as very fine wet sands, drilling muds, and so forth.

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52. Silver, Francis, "Storage and Handling of Pulverized Materials," Mechanical Engineering, September, 1951, Vol. 73, pp. 730-734.

53. Davis, E.A. and Pottberg, Rolfe, "Problems in Storing and Handling Pulverized Materials," Mechanical Engineering, March, 1952, Vol. 74, p.246.

Theory: Thixotropic materials become fluid when subjected to rapid agitation; but tend to segregate (freeze up) with gentle treatment.

Conclusion: The rapid, small amplitude treatment provided by vibration would be expected to be most effective, and neither aeration nor the displacement devices would be chosen."

SELECTION OF AN EXPERIMENTAL PROBLEM

In accordance with work simplification doctrine, the best way to do a job is to eliminate it. Perhaps this is the answer to flow stoppage in bins. Don't store the material in bins at all. By so doing, we would eliminate the wall-to-material friction, the high compaction factor and certainly circumvent the question of the value of the sloped side. We don't, however, reduce the materials handling problem. Even in its simplest storage on the open ground, bulk materials have an angle of repose, compaction, and moisture content and it is these three factors which are the major contributors to flow stoppage. Furthermore, open storage does not offer the demand flow of bins, so here we are back to bins. The only remaining approach, then, is to design bins to aid flow.

Bridging occurs through the ability of the material to transmit its weight to the side walls of the bin via the coefficient of friction of material to wall. The resolution of arch forces presented by Lee[54], Figure (15), page 41, illustrates how the force reaction of the retaining wall and the frictional drag are combined into a resultant lateral force which is inclined at the wall. The resulting force network is domelike, the center of curvature of which is defined. Lee[55] further rationalized that the combination of a vertical wall and an outwardly sloping wall (+ slope) would provide a dome with a shorter radius of curvature thereby being a stronger arch. If this is true, then consider the effect of having a bin with inwardly sloping walls (- slope) as shown in Figure (29). As the wall is sloped inward the resultant of the wall force and drag force

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54. Ibid. 1

55. Idem.

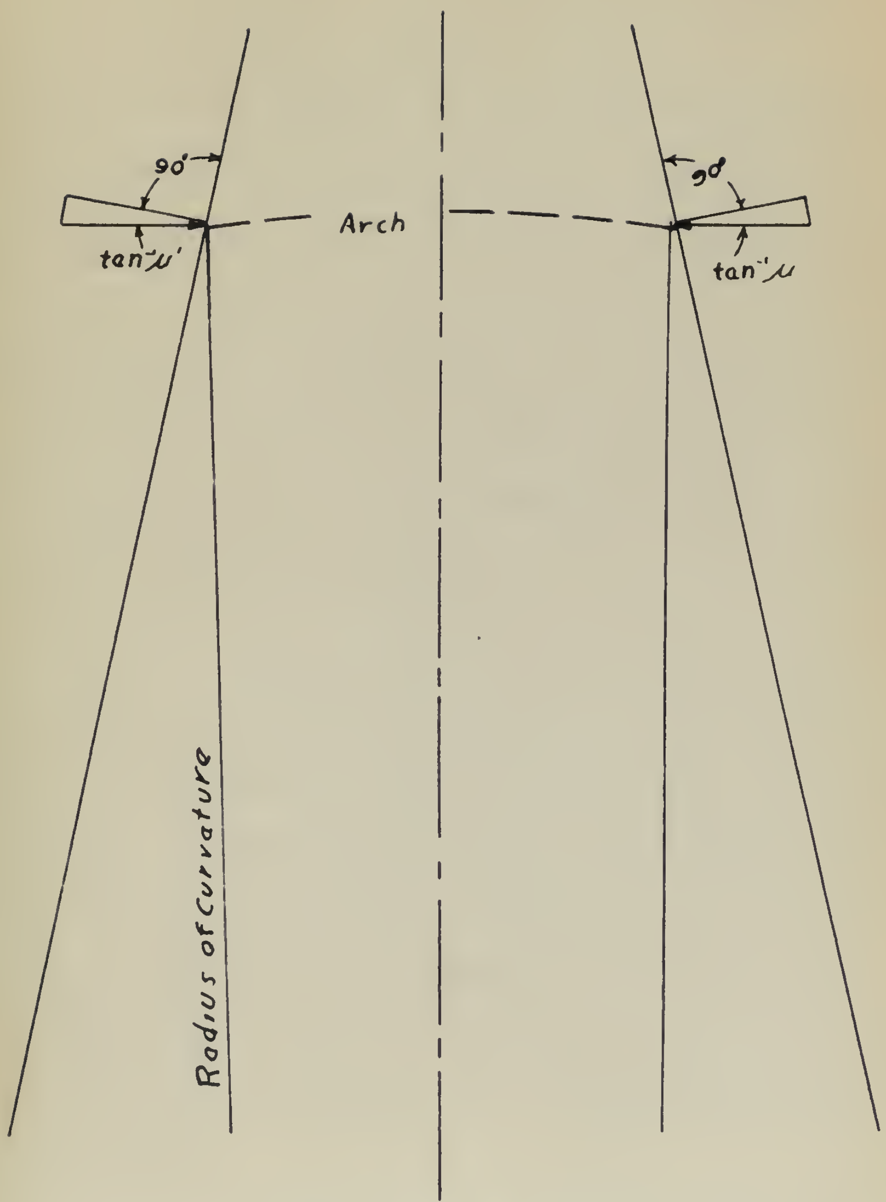

Fig. 29

Arch System of Pressure Forces

For Inward Sloping Walls.

would approach the horizontal. The resultant of the wall force and drag force approaching the horizontal would then develop an arch of infinite radius of curvature or of minimum strength. The hypothesis, therefore, is that a negatively-sloped bin would have less arch supporting action as the angle increases.

Of course one must realize that as the walls are inclined negatively the storage volume decreases. The question now to be considered is, "What is the proportionate increase in flowability for a given decrease in storage volume?" To answer this question a series of experiments were conducted to determine what relationship, if any, existed between the slope of the upper bin walls and the flowability of materials. It is anticipated that a noticeable volume discharge will be obtained by using a bin with a slight negative angle.

Test Procedure

A free-flowing and a cohesive material were separately handled in a test bin to determine the effect of slope angle upon the percent volume discharge. The bin (See Figure (30)) was a wooden structure two sides of which could be varied in inclination. The front wall was of plate glass to provide the opportunity to observe the particle flow and to record the flow by motion picture. The bottom was horizontal with a centrally located discharge area. A hopper bottom was not included in this design.

It is the feeling of the author that there are two independent sections in a bin: the upper section, which predominately has vertical sides and the hopper section. Flow stoppage was often defined as occurring at the transition of the vertical section to hopper section.

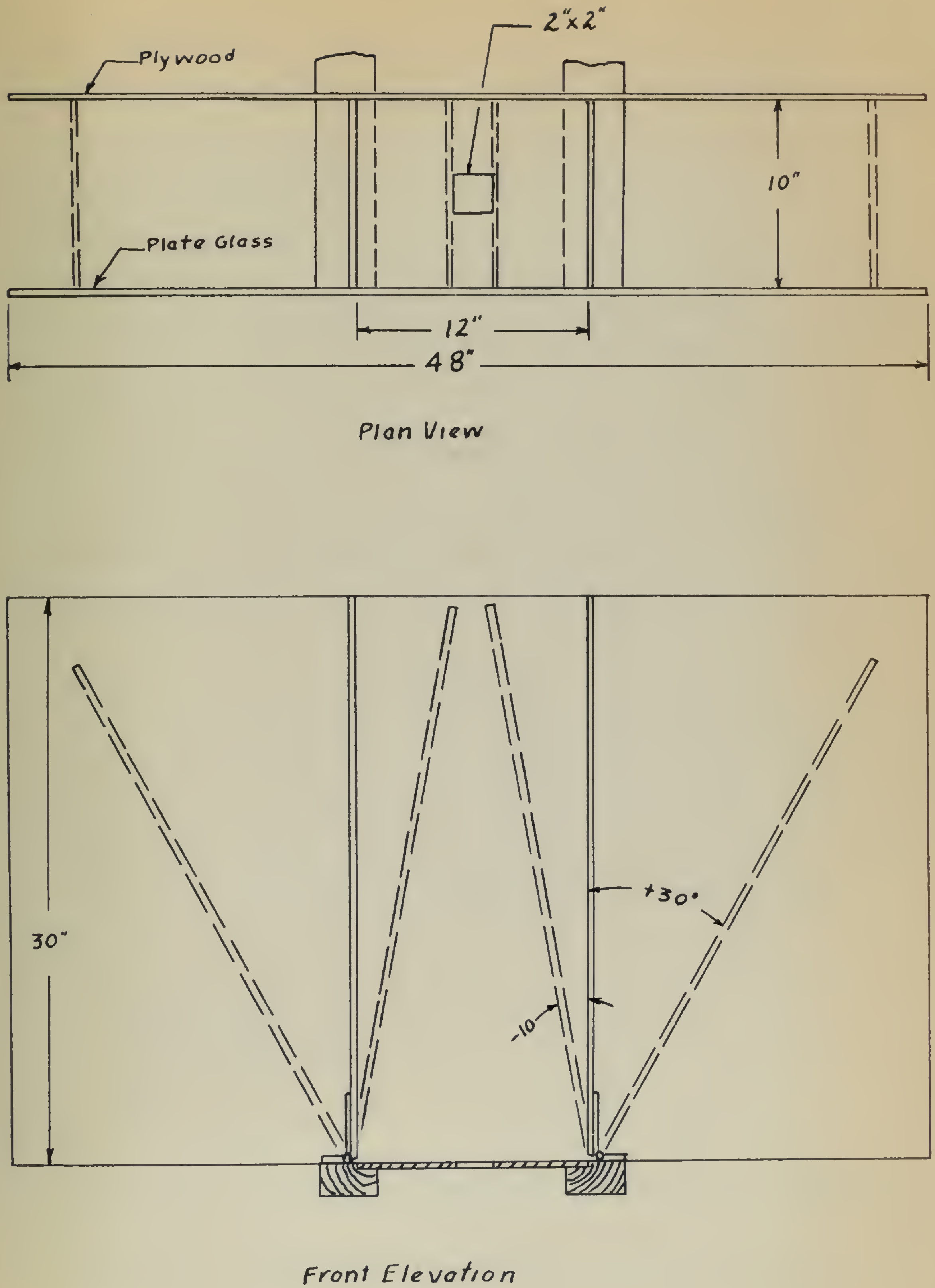

Fig. 30

Assembly of Test Bin

At this point the material has not undergone the excessive compaction that results from the wedge of the hopper. Assuming, therefore, that partial or whole discharge has occurred in the hopper, the major volume of the bin is being retained by continuous arches transmitting the weight to the side walls. If it is possible to reduce the transmission to side walls, the load of the material will be redistributed from the lateral transmission to a vertical transmission. It is anticipated that this load would then overcome the strength of the material and destroy the arch. The experiments then were designed to study the effect of side wall inclination upon the ability to arch. (The independence of the bin sections was first introduced by Leggett[56]. At the time this experiment was being conducted, Jenike's[57] paper had not been published.)

Bin Description

The bin was constructed of 3/8" plywood finished with an aluminized lacquer. The front wall of the bin was plate glass. The two opposing wood walls were pivoted at the bottom and had a range within the frame of $+30^\circ$ to -10°. The physical dimensions are as follows: (See Figure (30)).

Base Area (Internal)	10" x 12"
Discharge Area	2" x 2"
Base/Discharge Ratio	30 : 1
Height (Maximum)	30 inches
Head	24 inches
Slope Angle Range	$+30^\circ$ to -10°

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56. Leggett, R.F., "Clogging of Bituminous Coal in Bunkers," Trans. of the A.S.M.E., July, 1947, pp. 525-533.

57. Jenike, A.W., "Flow of Solids in Bulk Handling Systems", Feb. 1954. Unpublished. Salt Lake City, Utah.

The first phase of the test was conducted using cracked soybean as the free-flowing material. The soybean was loaded into the bin with the discharge closed. The angle of inclination of the adjustable side walls had been previously fixed. Upon completion of loading to the constant head height, the discharge was opened and the material allowed to flow free. The time of discharge was recorded and the amount of material which discharged was weighed. The angle of inclination (by definition the internal angle) of the material which was retained in the bin, was measured and recorded. The internal angle was recorded in the direction of the major axis (value a) and the minor axis (value b) to determine if there is a correlation with the proximity of the discharge area to the side wall. The bin was then completely discharged and the total mass of material originally contained in the bin was weighed and recorded. This process was repeated for each of the slope angles tested. (+ 30°, $+20^\circ$, $+10^\circ$, 0°, -5°, -10°). The average percent volume discharge and average discharge time was determined for each angle.

The controls maintained throughout the test were constant head, constant discharge area and record of moisture content. The moisture content was obtained for each series of the tests on a Cenco Moisture Balance using a 125-watt infra-red bulb with an exposure time of 9 minutes. The possibility that the material would abrade with usage was recognized. An attempt was made to minimize this occurrence by selecting an initial test volume twice as large as the capacity of the bin at maximum expansion. This constituted approximately four times the average amount required. After each series of tests, the entire mass was mulled together and another specimen taken. To determine if appreci-

able degradation was occurring, the angle of repose was measured at intervals during the test. The angle of repose is a function of the particle size and it was felt that any severe change would be highlighted by this test. The apparatus used to measure the angle of repose was a construction similar to that shown in Figure 1(a).

The flour was handled in the same sequence as that outlined for the soybean, except that flow was induced by jarring the bin. Once flow was induced, the bin was not disturbed again. The weight of the material discharged was recorded as was also the total mass contained in the bin. The internal slope angle was not recorded. An attempt was made to record the discharge time. However, in view of the required inducing of flow it was not felt that the time measurements were reliable.

Results

The results of the experiments as outlined are presented in Tables 2 and 3. The values tabulated are the average values for the slope angles considered. In the case of the results for the flour tests, only those angles listed were tested.

At first observation, one might conclude that the correlation of wall slope to percent discharge for the soybean is negative to that originally hypothesized. The percent volume discharge appears to decrease as the slope passes through the vertical to values of negative slope. A closer look at the data available, however, seems to indicate that there is little or no effect upon discharge as the result of sloping the side walls. This becomes more apparent when the material retained in the bin is considered. With the side walls vertical 7 pounds of material remained in the bin after flow had ceased. When the walls

TABLE 2

Experimental Data Obtained for Cracked Soybean

at Various Side Wall Inclinations

Slope Angle	Percent Volume Discharge	Time (minutes)	Weight Discharge	Capacity (pounds)	Internal Angle A	Internal Angle B
- 10°	86	0.39	39.3	45.5	40	38
- 5°	88	0.51	51.0	58.0	40.5	39
0°	90	0.59	63.5	70.5	40	39
+ 10	91	0.81	85.5	94	40	40
+ 20	92	1.07	115	125	38	39
+ 30	91	1.26	136	154	37	40

TABLE 3

Experimental Data Obtained

for

Flour at Various Side-Wall Inclinations

Slope Angle	Percentage Volume Discharge %	Time (minutes)	Weight Discharge (pounds)
-5°	41	xx	21
-0°	39	xx	23
$+10^\circ$	31	xx	24

were sloped ten degrees either side of the vertical, the material that remained was 6.2 pounds for -10° and 8.5 pounds for plus ten degrees. The change in retention in this case must be attributed to the change of capacity resulting from sloping the walls. The material slope is a function of the angle of repose and the wall friction. As the side walls were pivoted the retention volume changed slightly in the range referred to while the overall capacity of the bin changed considerably. It is felt that this differential in change of retention volume accounts for the change in per cent volume discharge. Therefore, the inclination of the upper bin wall has no effect upon the discharge of soybean. Soybean being a free-flowing material, it is felt that the same conclusion could be drawn for other free-flowing materials. Another significant feature to be considered is the rate of flow throughout the test. On an average the material flowed at the rate of one pound per .01 minute regardless of the side wall slope. This is in agreement with the conclusions of many that the discharge opening is the controlling factor of flow rate.

The cohesive material, flour, was not apparently affected by the slope of the side walls. First off, flow had to be induced in each test run. The flour was an ideal example of how difficult it is to maintain continuous flow in cohesive materials. As stated, it was necessary to induce flow. However, the flow ceased as soon as the plug of material immediately over the discharge left the bin. This left a perfect funnel: a funnel so stable it was possible to slice away the flour to the center cross-section with a spatula without destruction, as illustrated in Figure 31. The author feels this is a perfect

Fig. 31

Center-line Cross-section of the Funneling of flour

demonstration of funneling. Imagine the stability of a formation which will permit the removal of the front half without disturbing the remainder. The plug discharged was weighed in each instance. It is significant that there was little variance for the runs made at the three angles reported on. Regardless of the slope, the performance for every run was similar. When induced, the plug would flow and the same internal configuration occurred at all angles of inclination tested. It was felt at the time that the discharge area was again the controlling factor. Jenike's[58] work supported this contention. It may, therefore, be concluded that flour, a cohesive material, is not affected in its flow performance by the inclination of the upper bin walls.

In the description of the bin, it was stated that a glass front was provided for the purpose of taking motion pictures. Motion pictures were taken on runs of the various angles of inclination. This was extremely profitable for it recorded an observation made by the author in the early part of the work. In the illustration of centerline flow by Rudd[59] the center and top surface materials were in motion and there was an outside wedge of static material. In the test conducted by the author, it was observed that although the most dynamic flow was in the center, the material as far out as the wall was in an agitated state. Looking at the glass surface there was a mass movement of particles from a point seven inches above the base to the top of the bin. At the lower level, the dynamic area fanned out intersecting the side walls 11.5" above the base. A series of schematic diagrams presented in

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58. Ibid 57.

59. Ibid 12.

Figure (32) illustrates this flow pattern, as follows:

Figure 32(a) represents the material prior to discharge; figure 32(b) demonstrates the pattern defined; and figure 32(c) presents the retained material. The dense areas on either side of figure 32(c) are inclined at angle a. Although there were no dimensions on Rudd's[60] bin it may be classified as a shallow bin. The bin used by the author was designed to be a deep bin. The motion pattern defined may shed light on the flow pattern in deep bins. As the material discharged, there was a general subsiding of the mass until it reached the fan-out illustrated.

Conclusions

Based upon the bin and materials used in this experiment, the bin side has no apparent effect on material flow either for cohesive or free-flow materials. For free-flow materials the discharge area controls the flow rate. For cohesive materials the discharge area determines the ability to flow.

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60. Ibid 12.

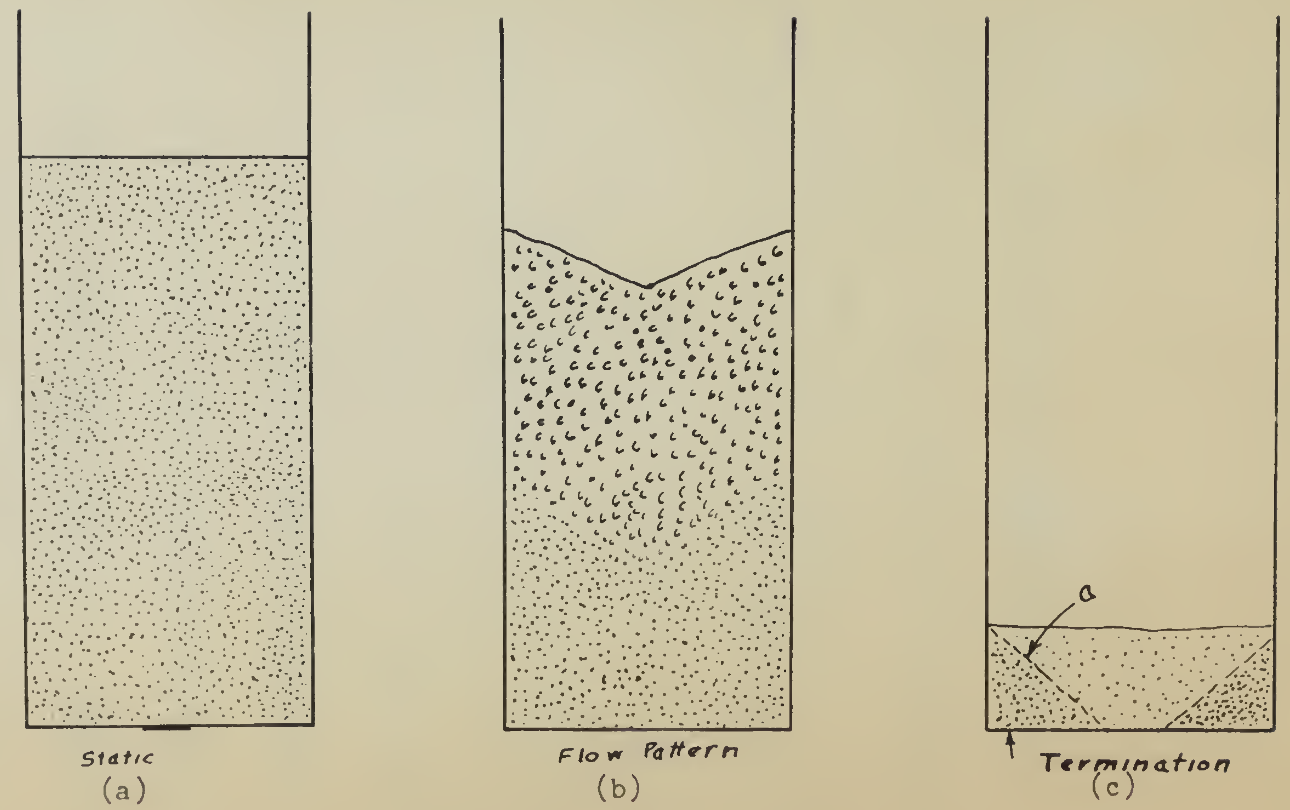

Fig. 32

Flow Pattern of Soybean Against Side Wall

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